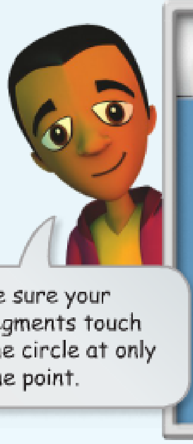
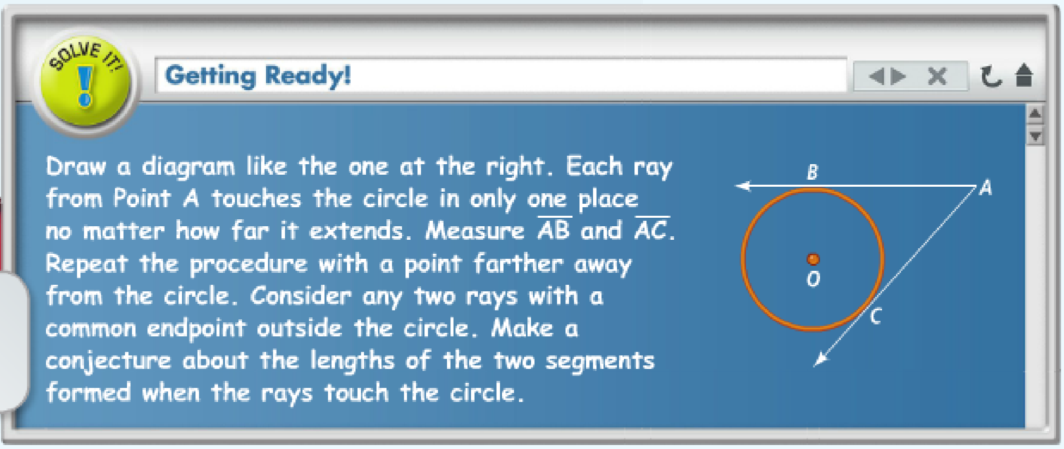
12-1 Tangent Lines

Objective To use properties of a tangent to a circle

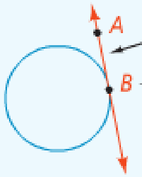

Be sure your segments touch the circle at only one point.



Draw a diagram like the one at the right. Each ray from Point A touches the circle in only one place no matter how far it extends. Measure $\overline{AB}$ and $\overline{AC}$. Repeat the procedure with a point farther away from the circle. Consider any two rays with a common endpoint outside the circle. Make a conjecture about the lengths of the two segments formed when the rays touch the circle.

In the Solve It, you drew lines that touch a circle at only one point. These lines are called tangents. This use of the word *tangent* is related to, but different from, the tangent ratio in right triangles that you studied in Chapter 8.

Lesson Vocabulary
- tangent to a circle
- point of tangency



A **tangent to a circle** is a line in the plane of the circle that intersects the circle in exactly one point.

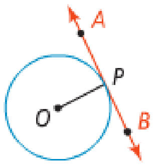
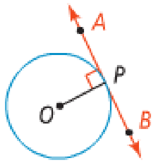
The point where a circle and a tangent intersect is the **point of tangency.**

$\overrightarrow{BA}$ is a tangent ray and $\overline{BA}$ is a tangent segment.

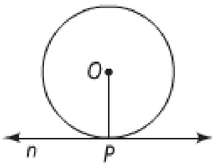
Essential Understanding A radius of a circle and the tangent that intersects the endpoint of the radius on the circle have a special relationship.

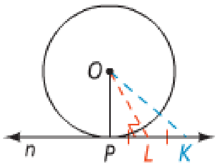
take note

Theorem 12-1

Theorem	If . . .	Then . . .
If a line is tangent to a circle, then the line is perpendicular to the radius at the point of tangency.	$\overleftrightarrow{AB}$ is tangent to $\odot O$ at P  	$\overleftrightarrow{AB} \perp \overline{OP}$ 

Proof Indirect Proof of Theorem 12-1





Given: n is tangent to $\odot O$ at P.

Prove: $n \perp \overline{OP}$

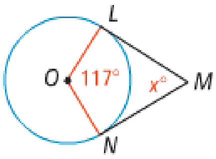
Step 1 Assume that n is not perpendicular to $\overline{OP}$.

Step 2 If line n is not perpendicular to $\overline{OP}$, then, for some other point L on n, $\overline{OL}$ must be perpendicular to n. Also there is a point K on n such that $\overline{LK} \cong \overline{LP}$. $\angle OLK \cong \angle OLP$ because perpendicular lines form congruent adjacent angles. $\overline{OL} \cong \overline{OL}$. So, $\triangle OLK \cong \triangle OLP$ by SAS.

Since corresponding parts of congruent triangles are congruent, $\overline{OK} \cong \overline{OP}$. So K and P are both on $\odot O$ by the definition of a circle. For two points on n to also be on $\odot O$ contradicts the given fact that n is tangent to $\odot O$ at P. So the assumption that n is not perpendicular to $\overline{OP}$ must be false.

Step 3 Therefore, $n \perp \overline{OP}$ must be true.

Problem 1 Finding Angle Measures



Multiple Choice $\overline{ML}$ and $\overline{MN}$ are tangent to $\odot O$. What is the value of x?

Ⓐ 58 Ⓒ 90

Ⓑ 63 Ⓓ 117

Think

What kind of angle is formed by a radius and a tangent?

The angle formed is a right angle, so the measure is 90.

Since $\overline{ML}$ and $\overline{MN}$ are tangent to $\odot O$, $\angle L$ and $\angle N$ are right angles. $LMNO$ is a quadrilateral. So the sum of the angle measures is 360.

$$m\angle L + m\angle M + m\angle N + m\angle O = 360$$

$$90 + m\angle M + 90 + 117 = 360 \quad \text{Substitute.}$$

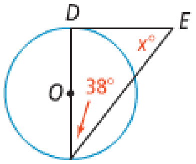
$$297 + m\angle M = 360 \quad \text{Simplify.}$$

$$m\angle M = 63 \quad \text{Solve.}$$

The correct answer is B.

Got It? **1. a.** $\overline{ED}$ is tangent to $\odot O$. What is the value of x?

b. Reasoning Consider a quadrilateral like the one in Problem 1. Write a formula you could use to find the measure of any angle x formed by two tangents when you know the measure of the central angle c whose radii intersect the tangents.



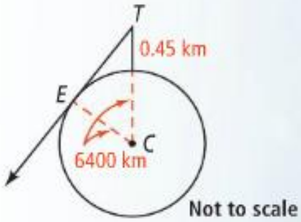
Problem 2 Finding Distance

Earth Science **The CN Tower in Toronto, Canada, has an observation deck 447 m above ground level. About how far is it from the observation deck to the horizon? Earth's radius is about 6400 km.**

Plan

How does knowing Earth's radius help?
The radius forms a right angle with a tangent line from the observation deck to the horizon. So, you can use two radii, the tower's height, and the tangent to form a right triangle.

Step 1 Make a sketch. The length 447 m is about 0.45 km.



Step 2 Use the Pythagorean Theorem.

$$\begin{aligned} CT^2 &= TE^2 + CE^2 \\ (6400 + 0.45)^2 &= TE^2 + 6400^2 && \text{Substitute.} \\ (6400.45)^2 &= TE^2 + 6400^2 && \text{Simplify.} \\ 40{,}965{,}760.2025 &= TE^2 + 40{,}960{,}000 && \text{Use a calculator.} \\ 5760.2025 &= TE^2 && \text{Subtract 40,960,000 from each side.} \\ 76 &\approx TE && \text{Take the positive square root of each side.} \end{aligned}$$

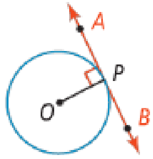
The distance from the CN Tower to the horizon is about 76 km.

Got It? **2.** What is the distance to the horizon that a person can see on a clear day from an airplane 2 mi above Earth? Earth's radius is about 4000 mi.

Theorem 12-2 is the converse of Theorem 12-1. You can use it to prove that a line or segment is tangent to a circle. You can also use it to construct a tangent to a circle.

take note

Theorem 12-2

Theorem	**If . . .**	**Then . . .**
If a line in the plane of a circle is perpendicular to a radius at its endpoint on the circle, then the line is tangent to the circle.	$\overleftrightarrow{AB} \perp \overline{OP}$ at P 	$\overleftrightarrow{AB}$ is tangent to $\odot O$

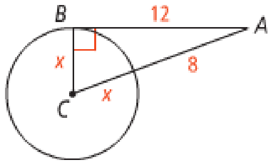
You will prove Theorem 12-2 in Exercise 30.

Problem 3 Finding a Radius

Think

Why does the value x appear on each side of the equation?
The length of $\overline{AC}$, the hypotenuse, is the radius plus 8, which is on the left side of the equation. On the right side of the equation, the radius is one side of the triangle.

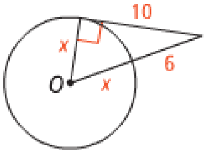
What is the radius of $\odot C$?



$$
\begin{aligned}
AC^2 &= AB^2 + BC^2 && \text{Pythagorean Theorem}\\
(x + 8)^2 &= 12^2 + x^2 && \text{Substitute.}\\
x^2 + 16x + 64 &= 144 + x^2 && \text{Simplify.}\\
16x &= 80 && \text{Subtract } x^2 \text{ and 64 from each side.}\\
x &= 5 && \text{Divide each side by 16.}
\end{aligned}
$$

The radius is 5.

Got It? **3.** What is the radius of $\odot O$?

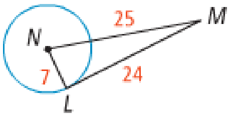


Problem 4 Identifying a Tangent

Think

What information does the diagram give you?
- LMN is a triangle.
- $NM = 25$, $LM = 24$, $NL = 7$
- $\overline{NL}$ is a radius.

Is $\overline{ML}$ tangent to $\odot N$ at L? Explain.



Know

The lengths of the sides of $\triangle LMN$

Need

To determine whether $\overline{ML}$ is tangent to $\odot O$

Plan

$\overline{ML}$ is a tangent if $\overline{ML} \perp \overline{NL}$. Use the Converse of the Pythagorean Theorem to determine whether $\triangle LMN$ is a right triangle.

$$
\begin{aligned}
NL^2 + ML^2 &\overset{?}{=} NM^2 &&\\
7^2 + 24^2 &\overset{?}{=} 25^2 && \text{Substitute.}\\
625 &= 625 && \text{Simplify.}
\end{aligned}
$$

By the Converse of the Pythagorean Theorem, $\triangle LMN$ is a right triangle with $\overline{ML} \perp \overline{NL}$. So $\overline{ML}$ is tangent to $\odot N$ at L because it is perpendicular to the radius at the point of tangency (Theorem 12-2).

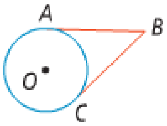
Got It? **4.** Use the diagram in Problem 4. If $NL = 4$, $ML = 7$, and $NM = 8$, is $\overline{ML}$ tangent to $\odot N$ at L? Explain.

In the Solve It, you made a conjecture about the lengths of two tangents from a common endpoint outside a circle. Your conjecture may be confirmed by the following theorem.

take note

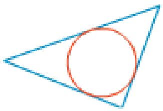
Theorem 12-3

Theorem	If . . .	Then . . .
If two tangent segments to a circle share a common endpoint outside the circle, then the two segments are congruent.	$\overline{BA}$ and $\overline{BC}$ are tangent to $\odot O$	$\overline{BA} \cong \overline{BC}$



You will prove Theorem 12-3 in Exercise 23.

In the figure at the right, the sides of the triangle are tangent to the circle. The circle is *inscribed in* the triangle. The triangle is *circumscribed about* the circle.

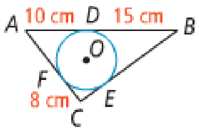


Problem 5 Circles Inscribed in Polygons

$\odot O$ is inscribed in $\triangle ABC$. What is the perimeter of $\triangle ABC$?

Plan

How can you find the length of $\overline{BC}$?
Find the segments congruent to $\overline{BE}$ and $\overline{EC}$. Then use segment addition.



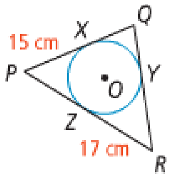
$AD = AF = 10$ cm
$BD = BE = 15$ cm
$CF = CE = 8$ cm

Two segments tangent to a circle from a point outside the circle are congruent, so they have the same length.

$p = AB + BC + CA$	Definition of perimeter p
$= AD + DB + BE + EC + CF + FA$	Segment Addition Postulate
$= 10 + 15 + 15 + 8 + 8 + 10$	Substitute.
$= 66$	

The perimeter is 66 cm.

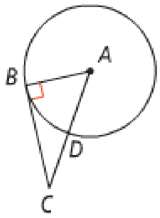
Got It? **5.** $\odot O$ is inscribed in $\triangle PQR$, which has a perimeter of 88 cm. What is the length of $\overline{QY}$?



Lesson Check

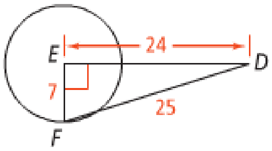
Do you know HOW?

1. If $m\angle A = 58$, what is $m\angle ACB$?
2. If $BC = 8$ and $DC = 4$, what is the radius?
3. If $AC = 12$ and $BC = 9$, what is the radius?



Do you UNDERSTAND?

4. **Vocabulary** How are the phrases *tangent ratio* and *tangent of a circle* used differently?
5. **Error Analysis** A classmate insists that $\overline{DF}$ is a tangent to $\odot E$. Explain how to show that your classmate is wrong.



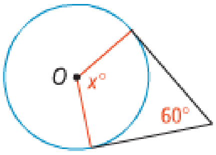
Practice and Problem-Solving Exercises

A Practice

Algebra Lines that appear to be tangent are tangent. *O* is the center of each circle. What is the value of *x*?

See Problem 1.

6.



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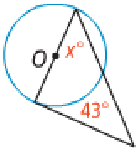
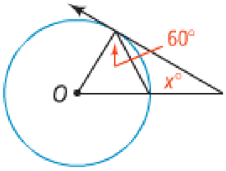
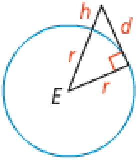


8.



Earth Science The circle at the right represents Earth. The radius of Earth is about 6400 km. Find the distance *d* to the horizon that a person can see on a clear day from each of the following heights *h* above Earth. Round your answer to the nearest tenth of a kilometer.



See Problem 2.

9. 5 km

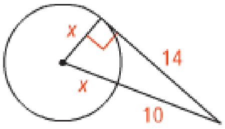
10. 1 km

11. 2500 m

Algebra In each circle, what is the value of *x*, to the nearest tenth?

See Problem 3.

12.



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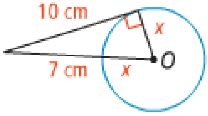

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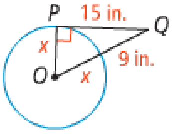
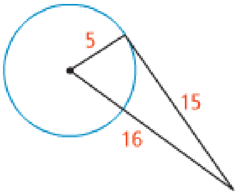


Determine whether a tangent is shown in each diagram. Explain.

See Problem 4.

15.



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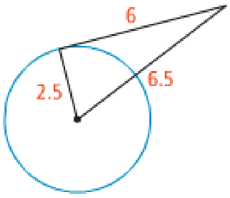

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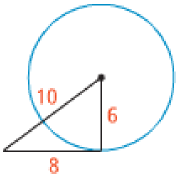
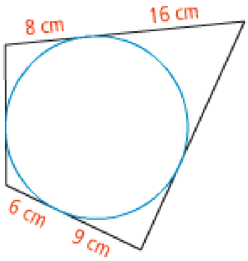


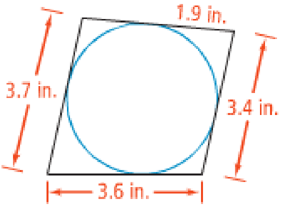
Each polygon circumscribes a circle. What is the perimeter of each polygon?

See Problem 5.

18.



19.

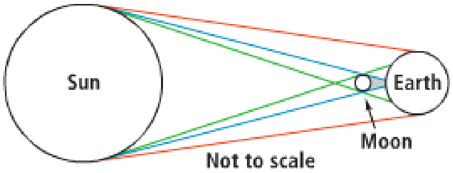


B Apply

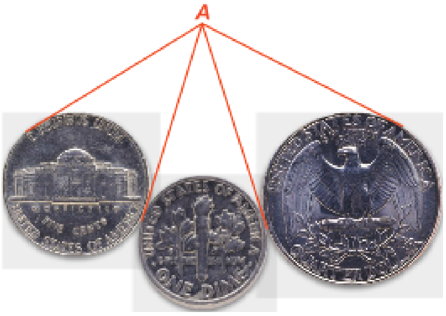
20. Solar Eclipse Common tangents to two circles may be *internal* or *external*. If you draw a segment joining the centers of the circles, a common internal tangent will intersect the segment. A common external tangent will not. For this cross-sectional diagram of the sun, moon, and Earth during a solar eclipse, use the terms above to describe the types of tangents of each color.

a. red **b.** blue **c.** green

d. Which tangents show the extent on Earth's surface of total eclipse? Of partial eclipse?



21. Reasoning A nickel, a dime, and a quarter are touching as shown. Tangents are drawn from point A to both sides of each coin. What can you conclude about the four tangent segments? Explain.



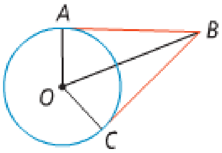
22. Think About a Plan Leonardo da Vinci wrote, "When each of two squares touch the same circle at four points, one is double the other." Explain why the statement is true.

- How will drawing a sketch help?
- Are both squares inside the circle?

23. Prove Theorem 12-3.
Proof

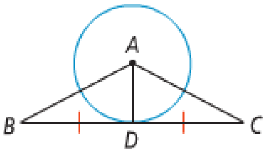
Given: $\overline{BA}$ and $\overline{BC}$ are tangent to $\odot O$ at A and C, respectively.

Prove: $\overline{BA} \cong \overline{BC}$



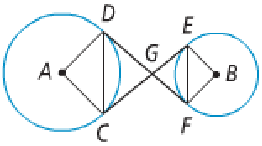
24. Given: $\overline{BC}$ is tangent to $\odot A$ at D.
Proof
$\overline{DB} \cong \overline{DC}$

Prove: $\overline{AB} \cong \overline{AC}$



25. Given: $\odot A$ and $\odot B$ with common tangents $\overline{DF}$ and $\overline{CE}$
Proof

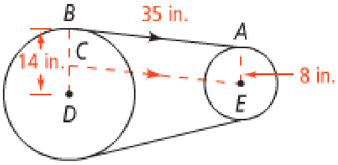
Prove: $\triangle GDC \sim \triangle GFE$



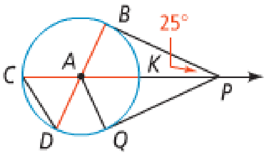
26. a. A belt fits snugly around the two circular pulleys. $\overline{CE}$ is an auxiliary line from E to $\overline{BD}$. $\overline{CE} \parallel \overline{BA}$. What type of quadrilateral is $ABCE$? Explain.

b. What is the length of $\overline{CE}$?

c. What is the distance between the centers of the pulleys to the nearest tenth?



27. $\overline{BD}$ and $\overline{CK}$ at the right are diameters of $\odot A$. $\overline{BP}$ and $\overline{QP}$ are tangents to $\odot A$. What is $m\angle CDA$?



28. Constructions Draw a circle. Label the center T. Locate a point on the circle and label it R. Construct a tangent to $\odot T$ at R.

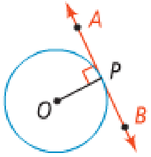
29. Coordinate Geometry Graph the equation $x^2 + y^2 = 9$. Then draw a segment from (0, 5) tangent to the circle. Find the length of the segment.

Challenge

30. Write an indirect proof of Theorem 12-2.

Proof **Given:** $\overline{AB} \perp \overline{OP}$ at P.

Prove: $\overline{AB}$ is tangent to $\odot O$.



31. Two circles that have one point in common are *tangent circles*. Given any triangle, explain how to draw three circles that are centered at each vertex of the triangle and are tangent to each other.

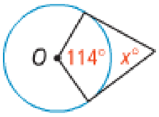
Standardized Test Prep

GRIDDED RESPONSE

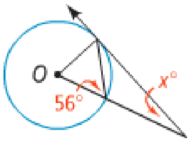
SAT/ACT

Lines in $\odot O$ that appear to be tangent are tangent. What is the value of x?

32.

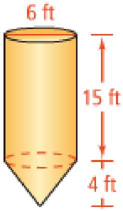


33.



34. The diagram at the right shows the dimensions of a silo. What is the volume of the silo to the nearest cubic foot?

35. The perimeter of an equilateral triangle is 90 in. What is its area to the nearest square inch?



Mixed Review

Two cubes have heights 6 in. and 8 in. Find each ratio. **See Lesson 11-7.**

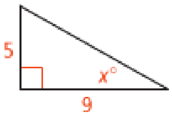
36. scale factor

37. ratio of surface areas

38. ratio of volumes

Algebra **Find the value of x. Round answers to the nearest tenth.** **See Lesson 8-3.**

39.



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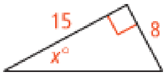

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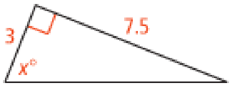
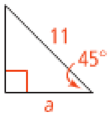


Get Ready! **To prepare for Lesson 12-2, do Exercises 42–44.**

Find the value of each variable. Leave your answer in simplest radical form. **See Lesson 8-2.**

42.



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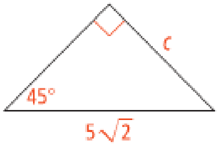

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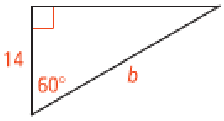


Concept Byte

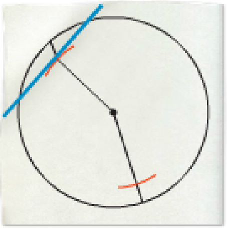
Use With Lesson 12-2

ACTIVITY

Paper Folding With Circles

A *chord* is a segment with endpoints on a circle. In these activities, you will explore some of the properties of chords.

Activity 1



Step 1 Use a compass. Draw a circle on tracing paper.

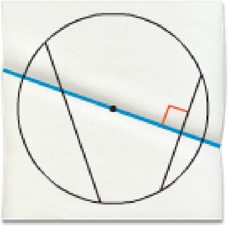
Step 2 Use a straightedge. Draw two radii.

Step 3 Set your compass to a distance shorter than the radii. Place its point at the center of the circle. Mark two congruent segments, one on each radius.

Step 4 Fold a line perpendicular to each radius at the point marked on the radius.

1. How do you measure the distance between a point and a line?
2. Each perpendicular contains a chord. Compare the lengths of the chords.
3. **Make a Conjecture** What is the relationship among the lengths of the chords that are equidistant from the center of a circle?

Activity 2



Step 1 Use a compass. Draw a circle on tracing paper.

Step 2 Use a straightedge. Draw two chords that are not diameters.

Step 3 Fold the perpendicular bisector for each chord.

4. Where do the perpendicular bisectors appear to intersect?
5. Draw a third chord and fold its perpendicular bisector. Where does it appear to intersect the other two?
6. **Make a Conjecture** What is true about the perpendicular bisector of a chord?

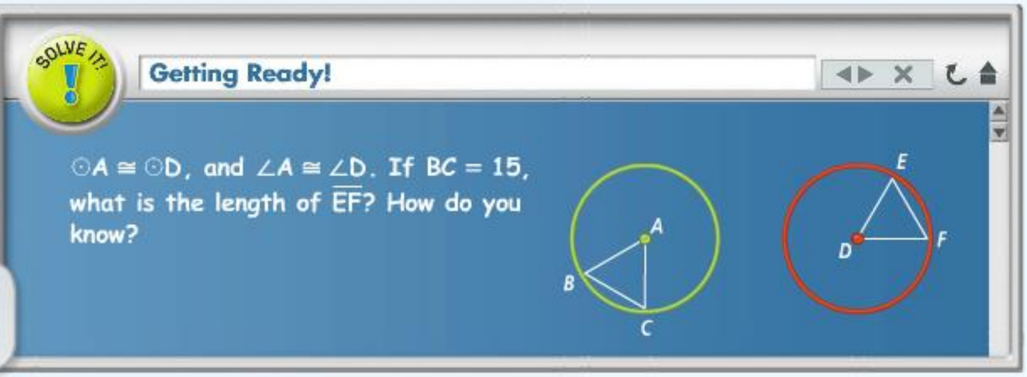
Exercises

7. Write a proof of your conjecture from Exercise 3 or give a counterexample.
8. What theorem provides a quick proof of your conjecture from Exercise 6?
9. **Make a Conjecture** Suppose two chords have different lengths. How do their distances from the center of the circle compare?
10. You are building a circular patio table. You have to drill a hole through the center of the tabletop for an umbrella. How can you find the center?

12-2 Chords and Arcs

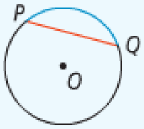
Objectives To use congruent chords, arcs, and central angles
To use perpendicular bisectors to chords

Congruent triangles come in handy, don't they?



Getting Ready!

$\odot A \cong \odot D$, and $\angle A \cong \angle D$. If $BC = 15$, what is the length of $\overline{EF}$? How do you know?



In the Solve It, you found the length of a **chord,** which is a segment whose endpoints are on a circle. The diagram shows the chord $\overline{PQ}$ and its related arc, $\overset{\frown}{PQ}$.

Dynamic Activity
Chords and Arcs

Lesson Vocabulary
- chord

Essential Understanding You can use information about congruent parts of a circle (or congruent circles) to find information about other parts of the circle (or circles).

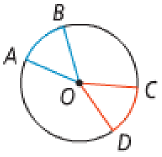
The following theorems and their converses confirm that if you know that chords, arcs, or central angles in a circle are congruent, then you know the other two parts are congruent.

take note

Theorem 12-4 and Its Converse

Theorem
Within a circle or in congruent circles, congruent central angles have congruent arcs.

Converse
Within a circle or in congruent circles, congruent arcs have congruent central angles.



If $\angle AOB \cong \angle COD$, then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.

If $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, then $\angle AOB \cong \angle COD$.

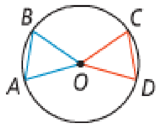
You will prove Theorem 12-4 and its converse in Exercises 19 and 35.

take note

Theorem 12-5 and Its Converse

Theorem
Within a circle or in congruent circles, congruent central angles have congruent chords.

Converse
Within a circle or in congruent circles, congruent chords have congruent central angles.



If $\angle AOB \cong \angle COD$, then $\overline{AB} \cong \overline{CD}$.

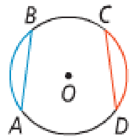
If $\overline{AB} \cong \overline{CD}$, then $\angle AOB \cong \angle COD$.

You will prove Theorem 12-5 and its converse in Exercises 20 and 36.

Theorem 12-6 and Its Converse

Theorem
Within a circle or in congruent circles, congruent chords have congruent arcs.

Converse
Within a circle or in congruent circles, congruent arcs have congruent chords.



If $\overline{AB} \cong \overline{CD}$, then $\widehat{AB} \cong \widehat{CD}$.

If $\widehat{AB} \cong \widehat{CD}$, then $\overline{AB} \cong \overline{CD}$.

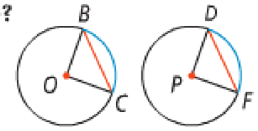
You will prove Theorem 12-6 and its converse in Exercises 21 and 37.

Problem 1 Using Congruent Chords

Think

Why is it important that the circles are congruent?
Two circles may have central angles with congruent chords, but the central angles will not be congruent unless the circles are congruent.

In the diagram, $\odot O \cong \odot P$. Given that $\overline{BC} \cong \overline{DF}$, what can you conclude?



$\angle O \cong \angle P$ because, within congruent circles, congruent chords have congruent central angles (conv. of Thm. 12-5). $\widehat{BC} \cong \widehat{DF}$ because, within congruent circles, congruent chords have congruent arcs (Thm. 12-6).

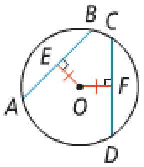
Got It? 1. Reasoning Use the diagram in Problem 1. Suppose you are given $\odot O \cong \odot P$ and $\angle OBC \cong \angle PDF$. How can you show $\angle O \cong \angle P$? From this, what else can you conclude?

take note

Theorem 12-7 and Its Converse

Theorem
Within a circle or in congruent circles, chords equidistant from the center or centers are congruent.

Converse
Within a circle or in congruent circles, congruent chords are equidistant from the center (or centers).

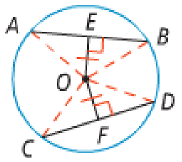


If $OE = OF$, then $\overline{AB} \cong \overline{CD}$.

If $\overline{AB} \cong \overline{CD}$, then $OE = OF$.

You will prove the converse of Theorem 12-7 in Exercise 38.

Proof Proof of Theorem 12-7



Given: $\odot O$, $\overline{OE} \cong \overline{OF}$, $\overline{OE} \perp \overline{AB}$, $\overline{OF} \perp \overline{CD}$

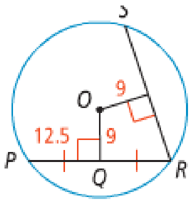
Prove: $\overline{AB} \cong \overline{CD}$

Statements	Reason
1) $\overline{OA} \cong \overline{OB} \cong \overline{OC} \cong \overline{OD}$	**1)** Radii of a circle are congruent.
2) $\overline{OE} \cong \overline{OF}$, $\overline{OE} \perp \overline{AB}$, $\overline{OF} \perp \overline{CD}$	**2)** Given
3) $\angle AEO$ and $\angle CFO$ are right angles.	**3)** Def. of perpendicular segments
4) $\triangle AEO \cong \triangle CFO$	**4)** HL Theorem
5) $\angle A \cong \angle C$	**5)** Corres. parts of ≅ △s are ≅.
6) $\angle B \cong \angle A$, $\angle C \cong \angle D$	**6)** Isosceles Triangle Theorem
7) $\angle B \cong \angle D$	**7)** Transitive Property of Congruence
8) $\angle AOB \cong \angle COD$	**8)** If two ∠s of a △ are ≅ to two ∠s of another △, then the third ∠s are ≅.
9) $\overline{AB} \cong \overline{CD}$	**9)** ≅ central angles have ≅ chords.

Problem 2 Finding the Length of a Chord

GRIDDED RESPONSE

What is the length of $\overline{RS}$ in $\odot O$?



Know

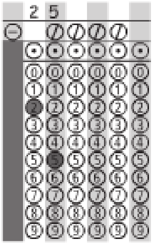
The diagram indicates that $PQ = QR = 12.5$ and $\overline{PR}$ and $\overline{RS}$ are both 9 units from the center.

Need

The length of chord $\overline{RS}$

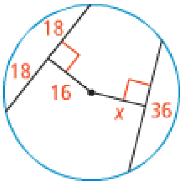
Plan

$\overline{PR} \cong \overline{RS}$, since they are the same distance from the center of the circle. So finding PR gives the length of $\overline{RS}$.



$PQ = QR = 12.5$	Given in the diagram
$PQ + QR = PR$	Segment Addition Postulate
$12.5 + 12.5 = PR$	Substitute.
$25 = PR$	Add.
$RS = PR$	Chords equidistant from the center of a circle are congruent.
$RS = 25$	Substitute.

Got It? **2.** What is the value of x? Justify your answer.



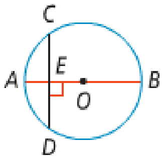
The Converse of the Perpendicular Bisector Theorem from Lesson 5-2 has special applications to a circle and its diameters, chords, and arcs.

take note

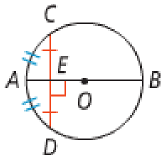
Theorem 12-8

Theorem
In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.

If . . .
$\overline{AB}$ is a diameter and $\overline{AB} \perp \overline{CD}$



Then . . .
$\overline{CE} \cong \overline{ED}$ and $\widehat{CA} \cong \widehat{AD}$

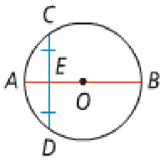


You will prove Theorem 12-8 in Exercise 22.

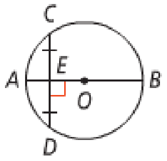
Theorem 12-9

Theorem
In a circle, if a diameter bisects a chord (that is not a diameter), then it is perpendicular to the chord.

If . . .
$\overline{AB}$ is a diameter and $\overline{CE} \cong \overline{ED}$



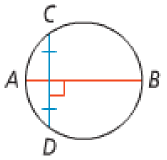
Then . . .
$\overline{AB} \perp \overline{CD}$



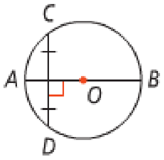
Theorem 12-10

Theorem
In a circle, the perpendicular bisector of a chord contains the center of the circle.

If . . .
$\overline{AB}$ is the perpendicular bisector of chord $\overline{CD}$

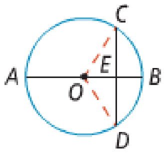


Then . . .
$\overline{AB}$ contains the center of $\odot O$



You will prove Theorem 12-10 in Exercise 33.

Proof Proof of Theorem 12-9



Given: $\odot O$ with diameter $\overline{AB}$ bisecting $\overline{CD}$ at E

Prove: $\overline{AB} \perp \overline{CD}$

Proof: $OC = OD$ because the radii of a circle are congruent. $CE = ED$ by the definition of bisect. Thus, O and E are both equidistant from C and D. By the Converse of the Perpendicular Bisector Theorem, both O and E are on the perpendicular bisector of $\overline{CD}$. Two points determine one line or segment, so $\overline{OE}$ is the perpendicular bisector of $\overline{CD}$. Since $\overline{OE}$ is part of $\overline{AB}$, $\overline{AB} \perp \overline{CD}$.

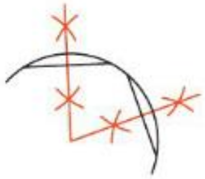
Problem 3 Using Diameters and Chords

Archaeology An archaeologist found pieces of a jar. She wants to find the radius of the rim of the jar to help guide her as she reassembles the pieces. What is the radius of the rim?

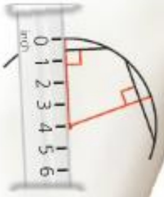
Think

How does the construction help find the center?
The perpendicular bisectors contain diameters of the circle. Two diameters intersect at the circle's center.

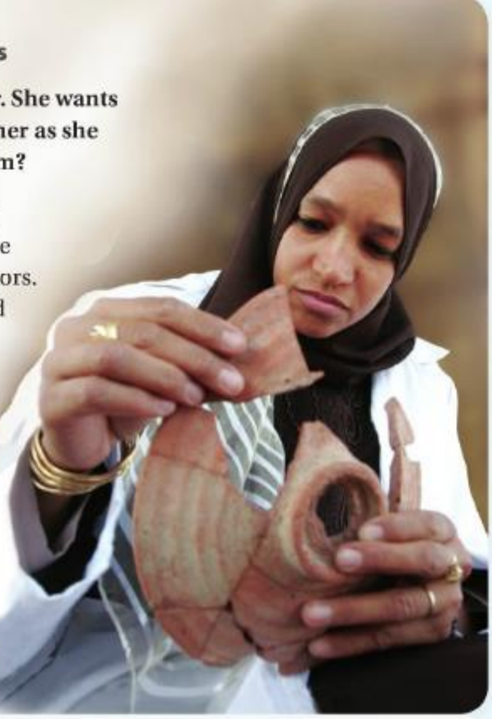
Step 1 Trace a piece of the rim. Draw two chords and construct perpendicular bisectors.



Step 2 The center is the intersection of the perpendicular bisectors. Use the center to find the radius.



The radius is 4 in.



Got It? **3.** Trace a coin. What is its radius?

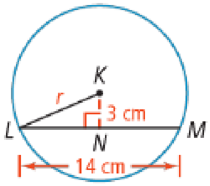
Problem 4 Finding Measures in a Circle

Algebra What is the value of each variable to the nearest tenth?

Plan

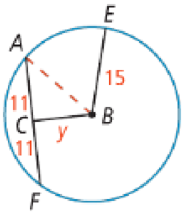
Find two sides of a right triangle. The third side is either the answer or leads to an answer.

A



$LN = \frac{1}{2}(14) = 7$	A diameter $\perp$ to a chord bisects the chord.
$r^2 = 3^2 + 7^2$	Use the Pythagorean Theorem.
$r \approx 7.6$	Find the positive square root of each side.

B



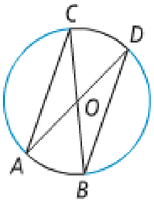
$\overline{BC} \perp \overline{AF}$	A diameter that bisects a chord that is not a diameter is $\perp$ to the chord.
$BA = BE = 15$	Draw an auxiliary $\overline{BA}$. The auxiliary $\overline{BA} \cong \overline{BE}$ because they are radii of the same circle.
$y^2 + 11^2 = 15^2$	Use the Pythagorean Theorem.
$y^2 = 104$	Solve for y^2.
$y \approx 10.2$	Find the positive square root of each side.

Got It? **4.** **Reasoning** In part (b), how does the auxiliary $\overline{BA}$ make the problem simpler to solve?

Lesson Check

Do you know HOW?

In $\odot O$, $m\overset{\frown}{CD} = 50$ and $\overline{CA} \cong \overline{BD}$.



1. What is $m\overset{\frown}{AB}$? How do you know?

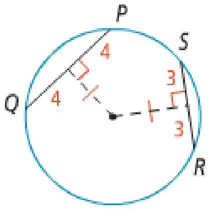
2. What is true of $\overset{\frown}{CA}$ and $\overset{\frown}{BD}$? Why?

3. Since $CA = BD$, what do you know about the distance of $\overline{CA}$ and $\overline{BD}$ from the center of $\odot O$?

Do you UNDERSTAND?

4. **Vocabulary** Is a radius a chord? Is a diameter a chord? Explain your answers.

5. **Error Analysis** What is the error in the diagram?



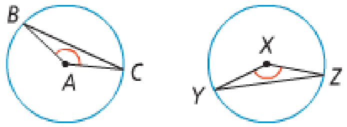
Practice and Problem-Solving Exercises

A Practice

In Exercises 6 and 7, the circles are congruent. What can you conclude?

See Problem 1.

6.



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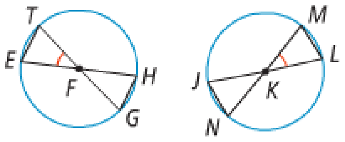
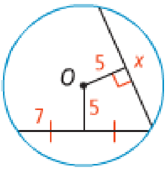


Find the value of x.

See Problem 2.

8.



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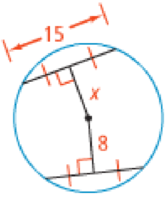

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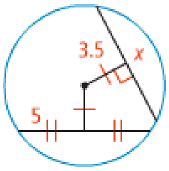
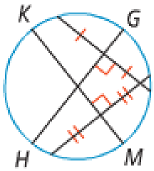
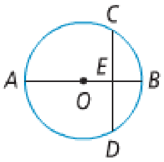


See Problems 3 and 4.

11. In the diagram at the right, $\overline{GH}$ and $\overline{KM}$ are perpendicular bisectors of the chords they intersect. What can you conclude about the center of the circle? Justify your answer.

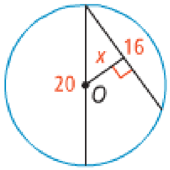


12. In $\odot O$, $\overline{AB}$ is a diameter of the circle and $\overline{AB} \perp \overline{CD}$. What conclusions can you make?



Algebra **Find the value of x to the nearest tenth.**

13.



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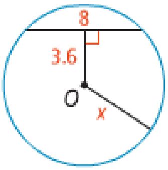
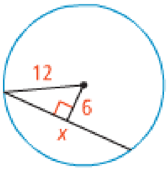
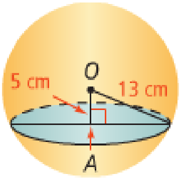


15.



B Apply

16. **Geometry in 3 Dimensions** In the figure at the right, sphere O with radius 13 cm is intersected by a plane 5 cm from center O. Find the radius of the cross section $\odot A$.



17. **Geometry in 3 Dimensions** A plane intersects a sphere that has radius 10 in., forming the cross section $\odot B$ with radius 8 in. How far is the plane from the center of the sphere?

18. **Think About a Plan** Two concentric circles have radii of 4 cm and 8 cm. A segment tangent to the smaller circle is a chord of the larger circle. What is the length of the segment to the nearest tenth?

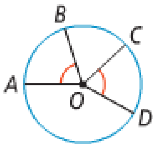
- How will you start the diagram?
- Where is the best place to position the radius of each circle?

19. Prove Theorem 12-4.

Proof

Given: $\odot O$ with $\angle AOB \cong \angle COD$

Prove: $\widehat{AB} \cong \widehat{CD}$

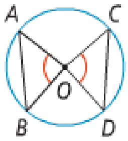


20. Prove Theorem 12-5.

Proof

Given: $\odot O$ with $\angle AOB \cong \angle COD$

Prove: $\overline{AB} \cong \overline{CD}$

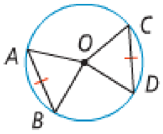


21. Prove Theorem 12-6.

Proof

Given: $\odot O$ with $\overline{AB} \cong \overline{CD}$

Prove: $\widehat{AB} \cong \widehat{CD}$

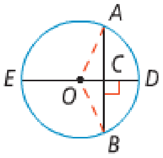


22. Prove Theorem 12-8.

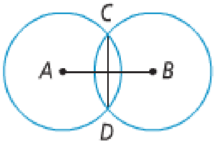
Proof

Given: $\odot O$ with diameter $\overline{ED} \perp \overline{AB}$ at C

Prove: $\overline{AC} \cong \overline{BC}$, $\widehat{AD} \cong \widehat{BD}$



$\odot A$ and $\odot B$ are congruent. $\overline{CD}$ is a chord of both circles.



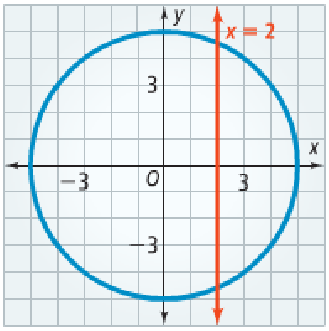
23. If $AB = 8$ in. and $CD = 6$ in., how long is a radius?

24. If $AB = 24$ cm and a radius $= 13$ cm, how long is $\overline{CD}$?

25. If a radius $= 13$ ft and $CD = 24$ ft, how long is $\overline{AB}$?

26. **Construction** Use Theorem 12-5 to construct a regular octagon.

27. In the diagram at the right, the endpoints of the chord are the points where the line $x = 2$ intersects the circle $x^2 + y^2 = 25$. What is the length of the chord? Round your answer to the nearest tenth.

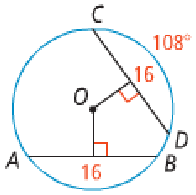


28. **Construction** Use a circular object such as a can or a saucer to draw a circle. Construct the center of the circle.

29. **Writing** Theorems 12-4 and 12-5 both begin with the phrase, "within a circle or in congruent circles." Explain why the word *congruent* is essential for both theorems.

Find $m\widehat{AB}$. (*Hint:* You will need to use trigonometry in Exercise 32.)

30.



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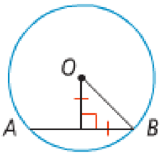
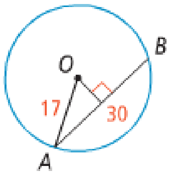


32.

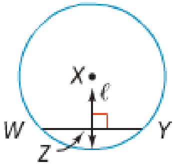


33. Prove Theorem 12-10.

Proof

Given: ℓ is the $\perp$ bisector of $\overline{WY}$.

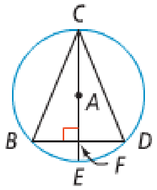
Prove: ℓ contains the center of $\odot X$.



34. **Given:** $\odot A$ with $\overline{CE} \perp \overline{BD}$

Proof

Prove: $\widehat{BC} \cong \widehat{DC}$



Challenge

Prove each of the following.

Proof

35. Converse of Theorem 12-4: Within a circle or in congruent circles, congruent arcs have congruent central angles.

36. Converse of Theorem 12-5: Within a circle or in congruent circles, congruent chords have congruent central angles.

37. Converse of Theorem 12-6: Within a circle or in congruent circles, congruent arcs have congruent chords.

38. Converse of Theorem 12-7: Within a circle or congruent circles, congruent chords are equidistant from the center (or centers).

39. If two circles are concentric and a chord of the larger circle is tangent to the smaller circle, prove that the point of tangency is the midpoint of the chord.

Proof

Standardized Test Prep

SAT/ACT

40. The diameter of a circle is 25 cm and a chord of the same circle is 16 cm. To the nearest tenth, what is the distance of the chord from the center of the circle?

(A) 9.0 cm
(B) 9.6 cm
(C) 18.0 cm
(D) 19.2 cm

41. The Smart Ball Company makes plastic balls for small children. The diameter of a ball is 8 cm. The cost for creating a ball is 2 cents per square centimeter. Which value is the most reasonable estimate for the cost of making 1000 balls?

(F) \$2010
(G) \$4021
(H) \$16,080
(I) \$42,900

42. From the top of a building you look down at an object on the ground. Your eyes are 50 ft above the ground and the angle of depression is 50°. Which distance is the best estimate of how far the object is from the base of the building?

(A) 42 ft
(B) 60 ft
(C) 65 ft
(D) 78 ft

Short Response

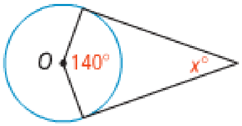
43. A bicycle tire has a diameter of 17 in. How many revolutions of the tire are necessary to travel 800 ft? Show your work.

Mixed Review

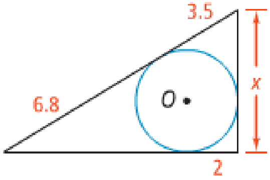
Assume that the lines that appear to be tangent are tangent. *O* is the center of each circle. Find the value of *x* to the nearest tenth.

See Lesson 12-1.

44.



45.



46. The legs of a right triangle are 10 in. and 24 in. long. The bisector of the right angle cuts the hypotenuse into two segments. What is the length of each segment, rounded to the nearest tenth?

See Lesson 7-5.

Get Ready! **To prepare for Lesson 12-3, do Exercises 47–52.**

See Lesson 10-6.

Identify the following in ⊙*P* at the right.

47. a semicircle

48. a minor arc

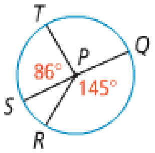
49. a major arc

Find the measure of each arc in ⊙*P*.

50. $\widehat{ST}$

51. $\widehat{STQ}$

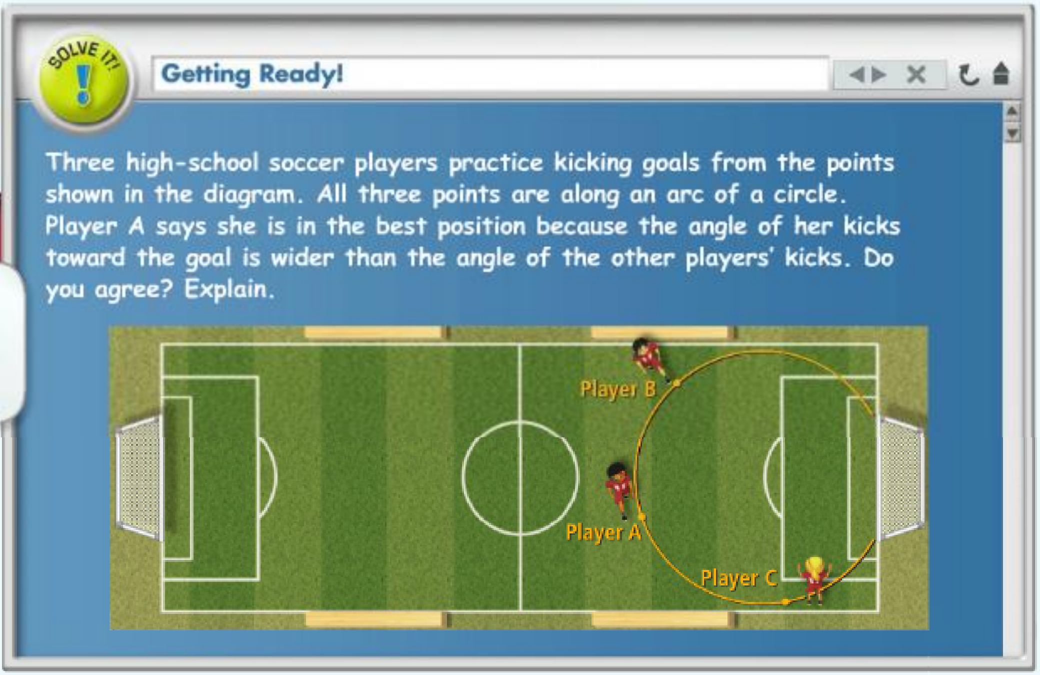
52. $\widehat{RT}$



12-3 Inscribed Angles

Objectives To find the measure of an inscribed angle
To find the measure of an angle formed by a tangent and a chord

Draw a large diagram and draw the angle each point makes with the goal posts.



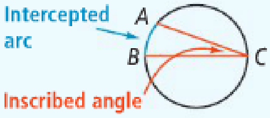
Getting Ready!

Three high-school soccer players practice kicking goals from the points shown in the diagram. All three points are along an arc of a circle. Player A says she is in the best position because the angle of her kicks toward the goal is wider than the angle of the other players' kicks. Do you agree? Explain.

Dynamic Activity
Inscribed Angles

Lesson Vocabulary
- inscribed angle
- intercepted arc

An angle whose vertex is on the circle and whose sides are chords of the circle is an **inscribed angle.** An arc with endpoints on the sides of an inscribed angle, and its other points in the interior of the angle is an **intercepted arc.** In the diagram, inscribed $\angle C$ intercepts $\widehat{AB}$.



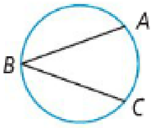
Essential Understanding Angles formed by intersecting lines have a special relationship to the arcs the intersecting lines intercept. In this lesson, you will study arcs formed by inscribed angles.

take note

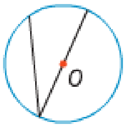
Theorem 12-11 Inscribed Angle Theorem

The measure of an inscribed angle is half the measure of its intercepted arc.

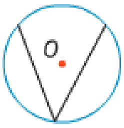
$$m\angle B = \frac{1}{2}m\widehat{AC}$$



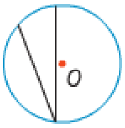
To prove Theorem 12-11, there are three cases to consider.



I: The center is on a side of the angle.



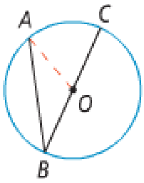
II: The center is inside the angle.



III: The center is outside the angle.

Below is a proof of Case I. You will prove Case II and Case III in Exercises 26 and 27.

Proof Proof of Theorem 12-11, Case I



Given: $\odot O$ with inscribed $\angle B$ and diameter $\overline{BC}$

Prove: $m\angle B = \frac{1}{2}m\overset{\frown}{AC}$

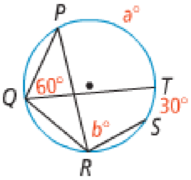
Draw radius $\overline{OA}$ to form isosceles $\triangle AOB$ with $OA = OB$ and, hence, $m\angle A = m\angle B$ (Isosceles Triangle Theorem).

$m\angle AOC = m\angle A + m\angle B$	Triangle Exterior Angle Theorem
$m\overset{\frown}{AC} = m\angle AOC$	Definition of measure of an arc
$m\overset{\frown}{AC} = m\angle A + m\angle B$	Substitute.
$m\overset{\frown}{AC} = 2m\angle B$	Substitute and simplify.
$\frac{1}{2}m\overset{\frown}{AC} = m\angle B$	Divide each side by 2.

Problem 1 Using the Inscribed Angle Theorem

Plan

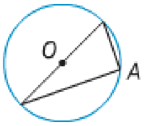
Which variable should you solve for first? You know the inscribed angle that intercepts $\overset{\frown}{PT}$, which has the measure a. You need a to find b. So find a first.



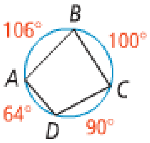
What are the values of a and b?

$m\angle PQT = \frac{1}{2}m\overset{\frown}{PT}$	Inscribed Angle Theorem
$60 = \frac{1}{2}a$	Substitute.
$120 = a$	Multiply each side by 2.
$m\angle PRS = \frac{1}{2}m\overset{\frown}{PS}$	Inscribed Angle Theorem
$m\angle PRS = \frac{1}{2}(m\overset{\frown}{PT} + m\overset{\frown}{TS})$	Arc Addition Postulate
$b = \frac{1}{2}(120 + 30)$	Substitute.
$b = 75$	Simplify.

Got It? **1. a.** In $\odot O$, what is $m\angle A$?



b. What are $m\angle A$, $m\angle B$, $m\angle C$, and $m\angle D$?



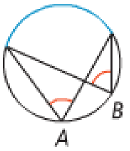
c. What do you notice about the sums of the measures of the opposite angles in the quadrilateral in part (b)?

You will use three corollaries to the Inscribed Angle Theorem to find measures of angles in circles. The first corollary may confirm an observation you made in the Solve It.

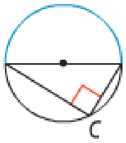
take note

Corollaries to Theorem 12-11: The Inscribed Angle Theorem

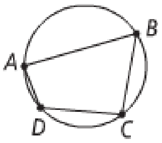
Corollary 1
Two inscribed angles that intercept the same arc are congruent.



Corollary 2
An angle inscribed in a semicircle is a right angle.



Corollary 3
The opposite angles of a quadrilateral inscribed in a circle are supplementary.



You will prove these corollaries in Exercises 31–33.

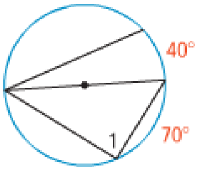
Think

Is there too much information?
Each diagram has more information than you need. Focus on what you need to find.

Problem 2 Using Corollaries to Find Angle Measures

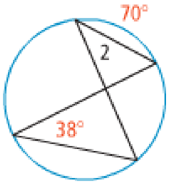
What is the measure of each numbered angle?

A



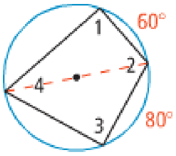
$\angle 1$ is inscribed in a semicircle. By Corollary 2, $\angle 1$ is a right angle, so $m\angle 1 = 90$.

B

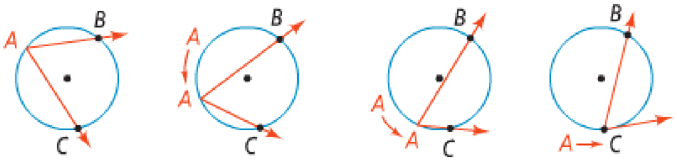


$\angle 2$ and the 38° angle intercept the same arc. By Corollary 1, the angles are congruent, so $m\angle 2 = 38$.

Got It? **2.** In the diagram at the right, what is the measure of each numbered angle?



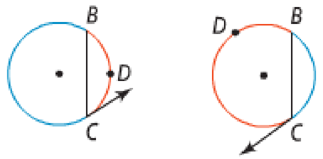
The following diagram shows point A moving along the circle until a tangent is formed. From the Inscribed Angle Theorem, you know that in the first three diagrams $m\angle A$ is $\frac{1}{2}m\widehat{BC}$. As the last diagram suggests, this is also true when A and C coincide.



take note

Theorem 12-12

The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc.

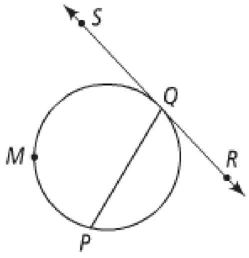


$$m\angle C = \frac{1}{2}m\overset{\frown}{BDC}$$

You will prove Theorem 12-12 in Exercise 34.

Problem 3 Using Arc Measure

In the diagram, $\overleftrightarrow{SR}$ is a tangent to the circle at Q. If $m\overset{\frown}{PMQ} = 212$, what is $m\angle PQR$?



Know

- $\overleftrightarrow{SR}$ is tangent to the circle at Q
- $m\overset{\frown}{PMQ} = 212$

Need

$m\angle PQR$

Plan

$m\angle PQS + m\angle PQR = 180$. So first find $m\angle PQS$ using $\overset{\frown}{PMQ}$.

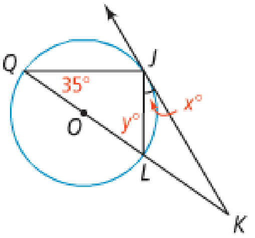
$\frac{1}{2}m\overset{\frown}{PMQ} = m\angle PQS$	The measure of an $\angle$ formed by a tangent and a chord is $\frac{1}{2}$ the measure of the intercepted arc.
$\frac{1}{2}(212) = m\angle PQS$	Substitute.
$106 = m\angle PQS$	Simplify.
$m\angle PQS + m\angle PQR = 180$	Linear Pair Postulate
$106 + m\angle PQR = 180$	Substitute.
$m\angle PQR = 74$	Simplify.

Think

How can you check the answer?
One way is to use $m\angle PQR$ to find $m\overset{\frown}{PQ}$. Confirm that $m\overset{\frown}{PQ} + m\overset{\frown}{PMQ} = 360$.

Got It? **3. a.** In the diagram at the right, $\overline{KJ}$ is tangent to $\odot O$. What are the values of x and y?

b. Reasoning In part (a), an inscribed angle ($\angle Q$) and an angle formed by a tangent and chord ($\angle KJL$) intercept the same arc. What is always true of these angles? Explain.



Lesson Check

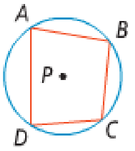
Do you know HOW?

Use the diagram for Exercises 1–3.

1. Which arc does $\angle A$ intercept?

2. Which angle intercepts $\overset{\frown}{ABC}$?

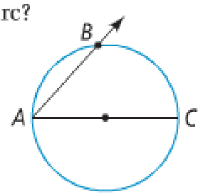
3. Which angles of quadrilateral $ABCD$ are supplementary?



Do you UNDERSTAND?

4. Vocabulary What is the relationship between an inscribed angle and its intercepted arc?

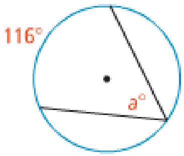
5. Error Analysis A classmate says that $m\angle A = 90$. What is your classmate's error?



Practice and Problem-Solving Exercises

A Practice **Find the value of each variable. For each circle, the dot represents the center.** See Problems 1 and 2.

6.



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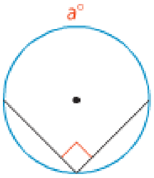

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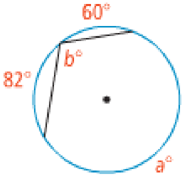

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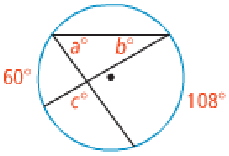

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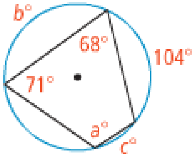
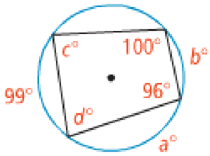


11.



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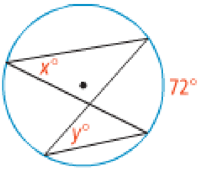

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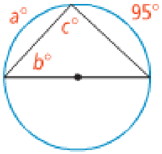

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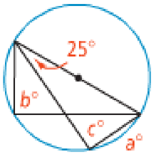

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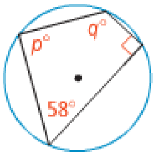
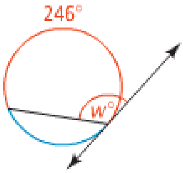


Find the value of each variable. Lines that appear to be tangent are tangent. See Problem 3.

16.



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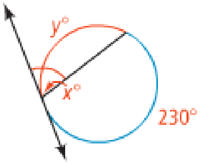

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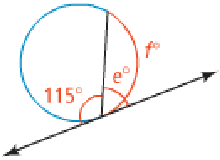
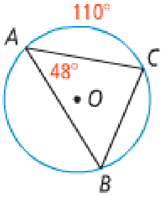


B Apply

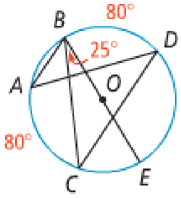
19. Writing A parallelogram inscribed in a circle must be what kind of parallelogram? Explain.

Find each indicated measure for $\odot O$.

20. **a.** $m\widehat{BC}$
b. $m\angle B$
c. $m\angle C$
d. $m\widehat{AB}$



21. **a.** $m\angle A$
b. $m\widehat{CE}$
c. $m\angle C$
d. $m\angle D$
e. $m\angle ABE$

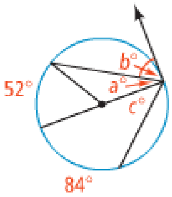


22. **Think About a Plan** What kind of trapezoid can be inscribed in a circle? Justify your response.

- Draw several diagrams to make a conjecture.
- How can parallel lines help?

Find the value of each variable. For each circle, the dot represents the center.

23.



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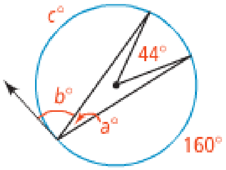

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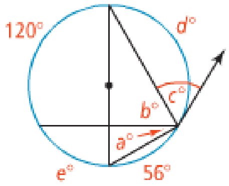
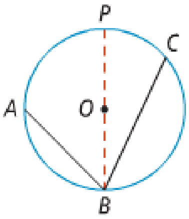


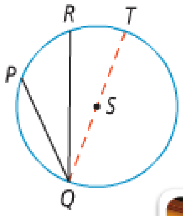
Write a proof for Exercises 26 and 27.

26. Inscribed Angle Theorem, Case II
Proof
Given: $\odot O$ with inscribed $\angle ABC$
Prove: $m\angle ABC = \frac{1}{2}m\widehat{AC}$



(*Hint:* Use the Inscribed Angle Theorem, Case I.)

27. Inscribed Angle Theorem, Case III
Proof
Given: $\odot S$ with inscribed $\angle PQR$
Prove: $m\angle PQR = \frac{1}{2}m\widehat{PR}$

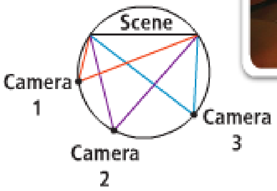


(*Hint:* Use the Inscribed Angle Theorem, Case I.)

28. **Television** The director of a telecast wants the option of showing the same scene from three different views.

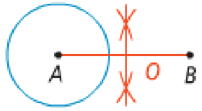
a. Explain why cameras in the positions shown in the diagram will transmit the same scene.

b. **Reasoning** Will the scenes look the same when the director views them on the control room monitors? Explain.

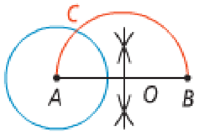


29. **Reasoning** Can a rhombus that is not a square be inscribed in a circle? Justify your answer.

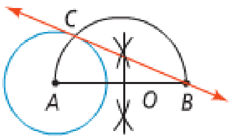
30. **Constructions** The diagrams below show the construction of a tangent to a circle from a point outside the circle. Explain why $\overleftrightarrow{BC}$ must be tangent to $\odot A$. (*Hint:* Copy the third diagram and draw $\overline{AC}$.)



Given: $\odot A$ and point B
Construct the midpoint of $\overline{AB}$. Label the point O.



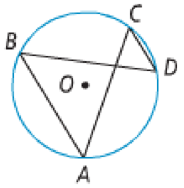
Construct a semicircle with radius OA and center O. Label its intersection with $\odot A$ as C.



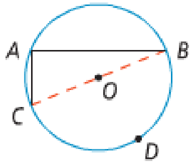
Draw $\overleftrightarrow{BC}$.

Write a proof for Exercises 31–34.

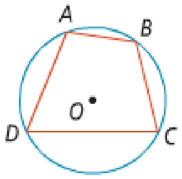
31. Inscribed Angle Theorem, Corollary 1
Proof
Given: $\odot O$, $\angle A$ intercepts $\widehat{BC}$, $\angle D$ intercepts $\widehat{BC}$.
Prove: $\angle A \cong \angle D$



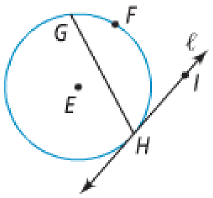
32. Inscribed Angle Theorem, Corollary 2
Proof
Given: $\odot O$ with $\angle CAB$ inscribed in a semicircle
Prove: $\angle CAB$ is a right angle.



33. Inscribed Angle Theorem, Corollary 3
Proof
Given: Quadrilateral $ABCD$ inscribed in $\odot O$
Prove: $\angle A$ and $\angle C$ are supplementary. $\angle B$ and $\angle D$ are supplementary.



34. Theorem 12-12
Proof
Given: $\overline{GH}$ and tangent ℓ intersecting $\odot E$ at H
Prove: $m\angle GHI = \frac{1}{2}m\widehat{GFH}$



Challenge

Reasoning **Is the statement *true* or *false*? If it is true, give a convincing argument. If it is false, give a counterexample.**

35. If two angles inscribed in a circle are congruent, then they intercept the same arc.

36. If an inscribed angle is a right angle, then it is inscribed in a semicircle.

37. A circle can always be circumscribed about a quadrilateral whose opposite angles are supplementary.

38. Prove that if two arcs of a circle are included between parallel chords, then the arcs are congruent.
Proof

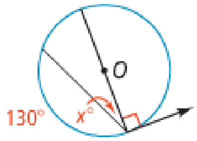
39. Constructions Draw two segments. Label their lengths x and y. Construct the geometric mean of x and y. (*Hint:* Recall a theorem about a geometric mean.)

Standardized Test Prep

SAT/ACT

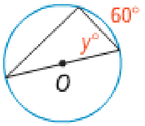
For Exercises 40 and 41, what is the value of each variable in ⊙O?

40.



Ⓐ 25
Ⓑ 35
Ⓒ 45
Ⓓ 65

41.



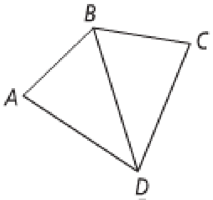
Ⓐ 20
Ⓑ 30
Ⓒ 50
Ⓓ 60

42. A large clock in a town square has a minute hand that is 10 ft long. Which value is the best estimate of how many degrees the minute hand moves from 4:00 to 4:40?

Ⓐ 40°　Ⓑ 120°　Ⓒ 240°　Ⓓ 400°

Extended Response

43. Is the following proof valid? If not, explain why, and then write a valid proof.



Given: Quadrilateral $ABCD$, $\angle A \cong \angle C$, $\overline{BD}$ bisects $\angle ABC$

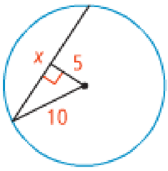
Prove: $\angle ADB \cong \angle CDB$

$\overline{BD} \cong \overline{BD}$ by the Reflexive Property. Since $\overline{BD}$ bisects $\angle ABC$, it also bisects $\angle ADC$. So $\angle ADB \cong \angle CDB$.

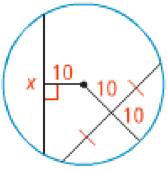
Mixed Review

Algebra **Find the value of x in ⊙O, to the nearest tenth.**　**See Lesson 12-2.**

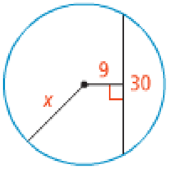
44.



45.



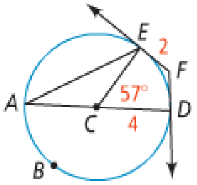
46.



47. The areas of two similar parallelograms are 20 cm^2 and 3.2 cm^2. What is the scale factor of the larger parallelogram to the smaller parallelogram?　**See Lesson 10-4.**

Get Ready! **To prepare for Lesson 12-4, do Exercises 48–51.**

In the diagram at the right, $\overrightarrow{FE}$ and $\overrightarrow{FD}$ are tangents to ⊙C at E and D, respectively.



See Lessons 12-1–12-3.

48. Find $m\widehat{DE}$.　**49.** Find $m\angle AEC$.

50. Find DF.　**51.** Find CE.

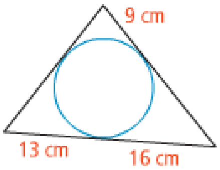
12 Mid-Chapter Quiz

MathXL® for School
Go to PowerGeometry.com

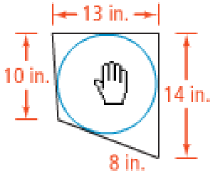
Do you know HOW?

Each polygon below circumscribes the circle. Find the perimeter of the polygon.

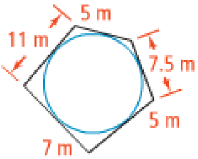
1.



2.



3.



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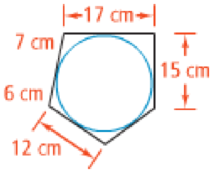
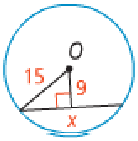
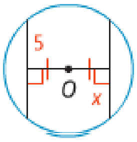


Algebra Find the value of x in ⊙O.

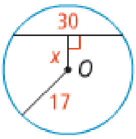
5.



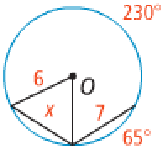
6.



7.

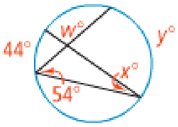


8.



Find the value of each variable. Lines that appear to be tangent are tangent, and the dot represents the center.

9.



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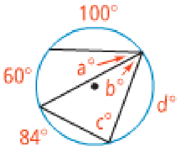

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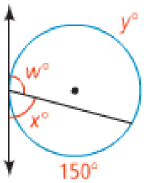

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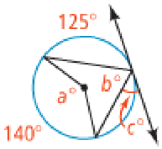
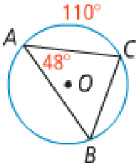


Find $m\widehat{AB}$.

13.



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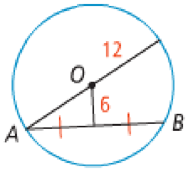
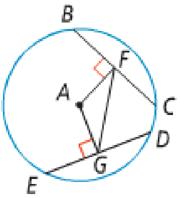


Write a two-column proof, paragraph proof, or flow proof.

15. Given: ⊙A with $\overline{BC} \cong \overline{DE}$, $\overline{AF} \perp \overline{BC}$, $\overline{AG} \perp \overline{DE}$

Proof

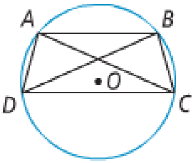
Prove: $\angle AFG \cong \angle AGF$



16. Given: ⊙O with $\widehat{AD} \cong \widehat{BC}$

Proof

Prove: $\triangle ABD \cong \triangle BAC$

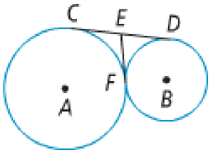


Do you UNDERSTAND?

17. Reasoning In ⊙C, $m\widehat{PQ} = 50$ and $m\widehat{QR} = 20$. Find two possible values for $m\widehat{PR}$.

18. Open-Ended Draw a triangle circumscribed about a circle. Then draw the radii to each tangent. How many convex quadrilaterals are in your figure?

19. Reasoning $\overline{EF}$ is tangent to both ⊙A and ⊙B at F. $\overline{CD}$ is tangent to ⊙A at C and to ⊙B at D. What can you conclude about $\overline{CE}$, $\overline{DE}$, and $\overline{FE}$? Explain.



20. Writing Explain why the length of a segment tangent to a circle from a point outside the circle will always be less than the distance from the point to the center of the circle.

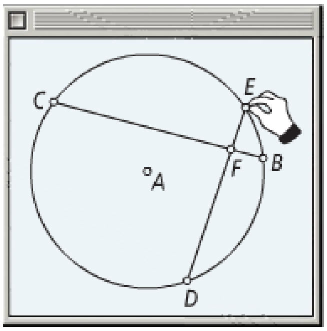
Concept Byte

Use With Lesson 12-4

TECHNOLOGY

Exploring Chords and Secants

Activity 1

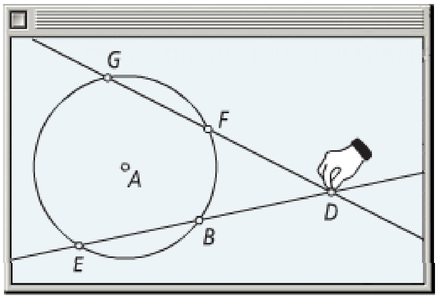


Construct ⊙A and two chords $\overline{BC}$ and $\overline{DE}$ that intersect at F.

1. Measure $\overline{BF}$, $\overline{FC}$, $\overline{EF}$, and $\overline{FD}$.
2. Use the calculator program of your software to find $BF \cdot FC$ and $EF \cdot FD$.
3. Manipulate the lines. What pattern do you observe in the products?
4. **Make a Conjecture** What appears to be true for two intersecting chords?

Activity 2

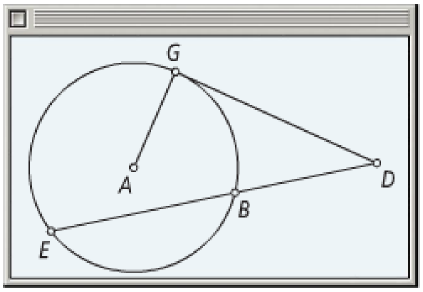
A *secant* is a line that intersects a circle in two points. A *secant segment* is a segment that contains a chord of the circle and has only one endpoint outside the circle. Construct a new circle and two secants $\overleftrightarrow{DG}$ and $\overleftrightarrow{DE}$ that intersect outside the circle at point D. Label the intersections with the circle as shown.



5. Measure $\overline{DG}$, $\overline{DF}$, $\overline{DE}$, and $\overline{DB}$.
6. Calculate the products $DG \cdot DF$ and $DE \cdot DB$.
7. Manipulate the lines. What pattern do you observe in the products?
8. **Make a Conjecture** What appears to be true for two intersecting secants?

Activity 3

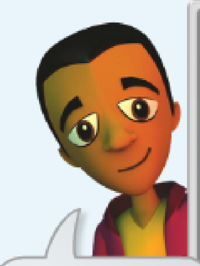
Construct ⊙A with tangent $\overline{DG}$ perpendicular to radius $\overline{AG}$ and secant $\overline{DE}$ that intersects the circle at B and E.



9. Measure $\overline{DG}$, $\overline{DE}$, and $\overline{DB}$.
10. Calculate the products $(DG)^2$ and $DE \cdot DB$.
11. Manipulate the lines. What pattern do you observe in the products?
12. **Make a Conjecture** What appears to be true for the tangent segment and secant segment?

12-4 Angle Measures and Segment Lengths

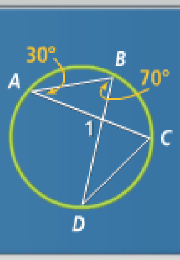
Objectives To find measures of angles formed by chords, secants, and tangents
To find the lengths of segments associated with circles



Think about how inscribed angles will help you out.

SOLVE IT! **Getting Ready!**

Find $m\angle 1$ and the sum of the measures of $\overset{\frown}{AD}$ and $\overset{\frown}{BC}$. What is the relationship between the measures? How do you know?



Lesson Vocabulary
- secant

Essential Understanding

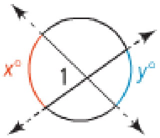
Essential Understanding Angles formed by intersecting lines have a special relationship to the related arcs formed when the lines intersect a circle. In this lesson, you will study angles and arcs formed by lines intersecting either within a circle or outside a circle.

take note

Theorem 12-13

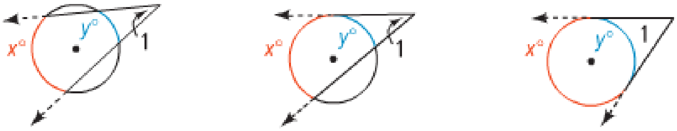
The measure of an angle formed by two lines that intersect inside a circle is half the sum of the measures of the intercepted arcs.

$$m\angle 1 = \frac{1}{2}(x + y)$$



Theorem 12-14

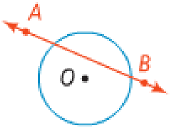
The measure of an angle formed by two lines that intersect outside a circle is half the difference of the measures of the intercepted arcs.



$$m\angle 1 = \frac{1}{2}(x - y)$$

You will prove Theorem 12-14 in Exercises 35 and 36.

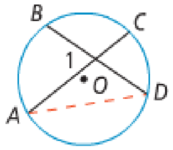
In Theorem 12-13, the lines from a point outside the circle going through the circle are called secants. A **secant** is a line that intersects a circle at two points. $\overleftrightarrow{AB}$ is a secant, $\overrightarrow{AB}$ is a secant ray, and $\overline{AB}$ is a secant segment. A chord is part of a secant.



Proof Proof of Theorem 12-13

Given: $\odot O$ with intersecting chords $\overline{AC}$ and $\overline{BD}$

Prove: $m\angle 1 = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$



Begin by drawing auxiliary $\overline{AD}$ as shown in the diagram.

$m\angle BDA = \frac{1}{2}m\widehat{AB}$, and $m\angle CAD = \frac{1}{2}m\widehat{CD}$

Inscribed Angle Theorem

$m\angle 1 = m\angle BDA + m\angle CAD$

△Exterior Angle Theorem

$m\angle 1 = \frac{1}{2}m\widehat{AB} + \frac{1}{2}m\widehat{CD}$

Substitute.

$m\angle 1 = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$

Distributive Property

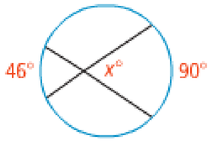
Problem 1 Finding Angle Measures

Think

Remember to add arc measures for arcs intercepted by lines that intersect inside a circle and subtract arc measures for arcs intercepted by lines that intersect outside a circle.

Algebra **What is the value of each variable?**

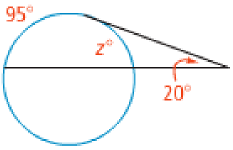
A



$x = \frac{1}{2}(46 + 90)$ Theorem 12-13

$x = 68$ Simplify.

B



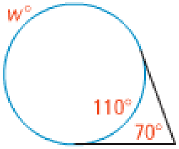
$20 = \frac{1}{2}(95 - z)$ Theorem 12-14

$40 = 95 - z$ Multiply each side by 2.

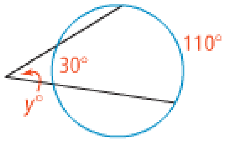
$z = 55$ Solve for z.

Got It? **1.** What is the value of each variable?

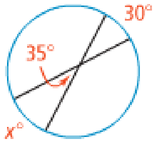
a.



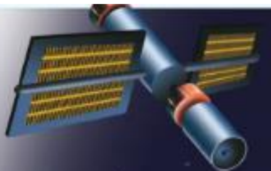
b.



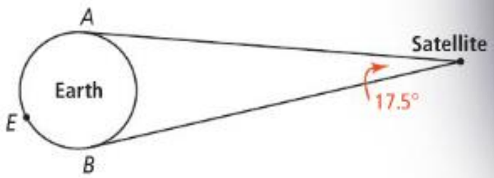
c.



Problem 2 Finding an Arc Measure



Satellite A satellite in a geostationary orbit above Earth's equator has a viewing angle of Earth formed by the two tangents to the equator. The viewing angle is about 17.5°. What is the measure of the arc of Earth that is viewed from the satellite?



Think

How can you represent the measures of the arcs? The sum of the measures of the arcs is 360°. If the measure of one arc is *x*, the measure of the other is 360 − *x*.

Let $m\widehat{AB} = x$.

Then $m\widehat{AEB} = 360 - x$.

$17.5 = \frac{1}{2}(m\widehat{AEB} - m\widehat{AB})$	Theorem 12-14
$17.5 = \frac{1}{2}[(360 - x) - x]$	Substitute.
$17.5 = \frac{1}{2}(360 - 2x)$	Simplify.
$17.5 = 180 - x$	Distributive Property
$x = 162.5$	Solve for *x*.

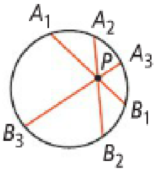
A 162.5° arc can be viewed from the satellite.

Got It? **2. a.** A departing space probe sends back a picture of Earth as it crosses Earth's equator. The angle formed by the two tangents to the equator is 20°. What is the measure of the arc of the equator that is visible to the space probe?

b. Reasoning Is the probe or the geostationary satellite in Problem 2 closer to Earth? Explain.

Essential Understanding There is a special relationship between two intersecting chords, two intersecting secants, or a secant that intersects a tangent. This relationship allows you to find the lengths of unknown segments.

From a given point *P*, you can draw two segments to a circle along infinitely many lines. For example, $\overline{PA_1}$ and $\overline{PB_1}$ lie along one such line. Theorem 12-15 states the surprising result that no matter which line you use, the product of the lengths $PA \cdot PB$ remains constant.

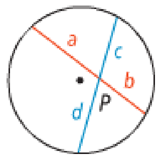


take note

Theorem 12-15

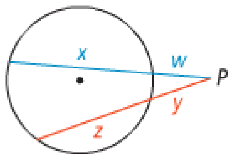
For a given point and circle, the product of the lengths of the two segments from the point to the circle is constant along any line through the point and circle.

I.



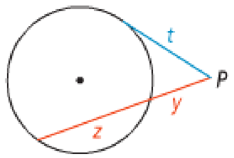
$a \cdot b = c \cdot d$

II.



$(w + x)w = (y + z)y$

III.



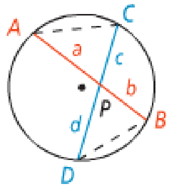
$(y + z)y = t^2$

As you use Theorem 12–15, remember the following.

- **Case I:** The products of the chord segments are equal.
- **Case II:** The products of the secants and their outer segments are equal.
- **Case III:** The product of a secant and its outer segment equals the square of the tangent.

Here is a proof for Case I. You will prove Case II and Case III in Exercises 37 and 38.

Proof Proof of Theorem 12-15, Case I



Given: A circle with chords $\overline{AB}$ and $\overline{CD}$ intersecting at P

Prove: $a \cdot b = c \cdot d$

Draw $\overline{AC}$ and $\overline{BD}$. $\angle A \cong \angle D$ and $\angle C \cong \angle B$ because each pair intercepts the same arc, and angles that intercept the same arc are congruent. $\triangle APC \sim \triangle DPB$ by the Angle-Angle Similarity Postulate. The lengths of corresponding sides of similar triangles are proportional, so $\frac{a}{d} = \frac{c}{b}$. Therefore, $a \cdot b = c \cdot d$.

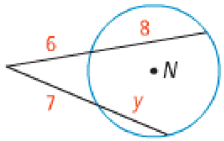
Problem 3 Finding Segment Lengths

Plan

How can you identify the segments needed to use Theorem 12-15? Find where segments intersect each other relative to the circle. The lengths of segments that are part of one line will be on the same side of an equation.

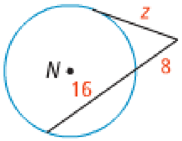
Algebra **Find the value of the variable in $\odot N$.**

A



$(6 + 8)6 = (7 + y)7$	Thm. 12-15, Case II
$84 = 49 + 7y$	Distributive Property
$35 = 7y$	
$5 = y$	Solve for y.

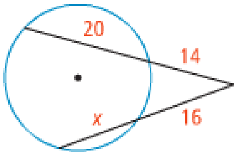
B



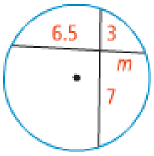
$(8 + 16)8 = z^2$	Thm. 12-15, Case III
$192 = z^2$	Simplify.
$13.9 \approx z$	Solve for z.

Got It? **3.** What is the value of the variable to the nearest tenth?

a.



b.



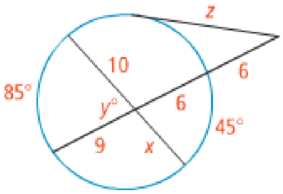
Lesson Check

Do you know HOW?

1. What is the value of x?

2. What is the value of y?

3. What is the value of z, to the nearest tenth?



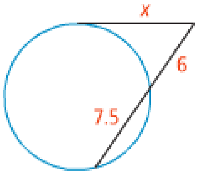
4. The measure of the angle formed by two tangents to a circle is 80. What are the measures of the intercepted arcs?

Do you UNDERSTAND?

5. Vocabulary Describe the difference between a *secant* and a *tangent*.

6. In the diagram for Exercises 1–3, is it possible to find the measures of the unmarked arcs? Explain.

7. Error Analysis To find the value of x, a student wrote the equation $(7.5)6 = x^2$. What error did the student make?



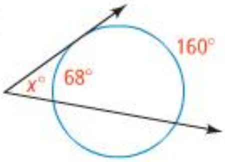
Practice and Problem-Solving Exercises

A Practice

Algebra Find the value of each variable.

See Problems 1 and 2.

8.



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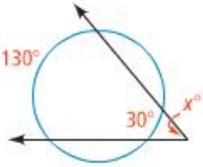

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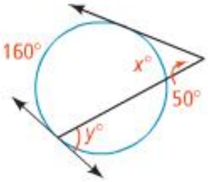

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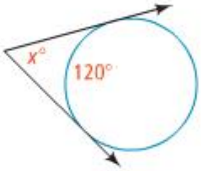
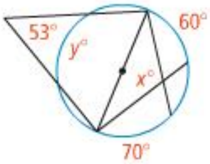
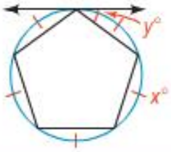


12.



13.

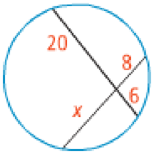


14. Photography You focus your camera on a circular fountain. Your camera is at the vertex of the angle formed by tangents to the fountain. You estimate that this angle is 40°. What is the measure of the arc of the circular basin of the fountain that will be in the photograph?

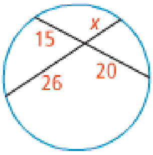
Algebra Find the value of each variable using the given chord, secant, and tangent lengths. If the answer is not a whole number, round to the nearest tenth.

See Problem 3.

15.



16.



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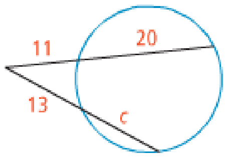

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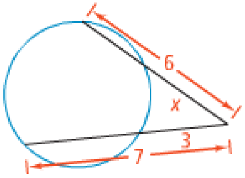
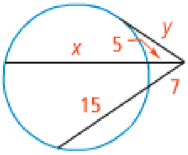
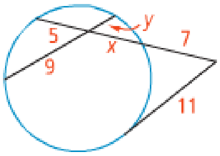


19.

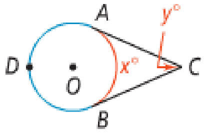


20.



B Apply

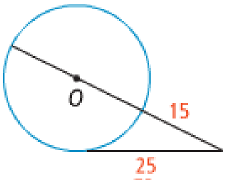
Algebra $\overline{CA}$ and $\overline{CB}$ are tangents to $\odot O$. Write an expression for each arc or angle in terms of the given variable.



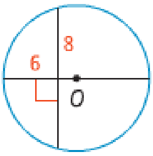
21. $m\widehat{ADB}$ using x **22.** $m\angle C$ using x **23.** $m\widehat{AB}$ using y

Find the diameter of $\odot O$. A line that appears to be tangent is tangent. If your answer is not a whole number, round it to the nearest tenth.

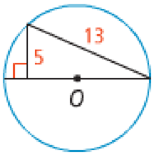
24.



25.

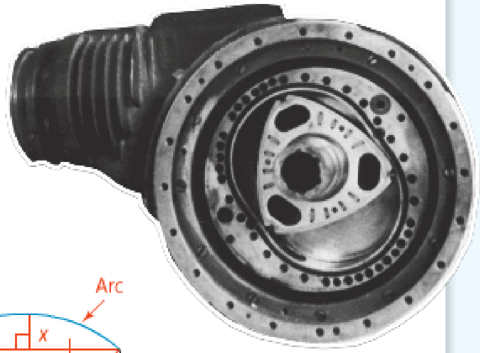


26.



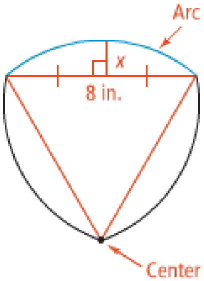
27. A circle is inscribed in a quadrilateral whose four angles have measures 85, 76, 94, and 105. Find the measures of the four arcs between consecutive points of tangency.

Wankel engine



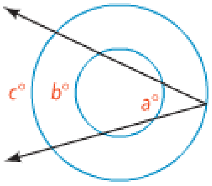
28. Engineering The basis for the design of the Wankel rotary engine is an equilateral triangle. Each side of the triangle is a chord to an arc of a circle. The opposite vertex of the triangle is the center of the circle that forms the arc. In the diagram below, each side of the equilateral triangle is 8 in. long.

- **a.** Use what you know about equilateral triangles and find the value of x.
- **b. Reasoning** Copy the diagram and complete the circle with the given center. Then use Theorem 12-15 to find the value of x. Show that your answers to parts (a) and (b) are equal.



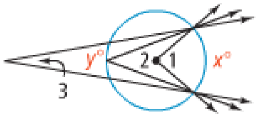
29. **Think About a Plan** In the diagram, the circles are concentric. What is a formula you could use to find the value of c in terms of a and b?

- How can you use the inscribed angle to find the value of c?
- What is the relationship of the inscribed angle to a and b?



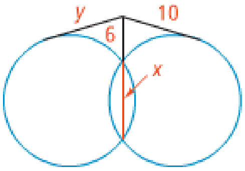
30. $\triangle PQR$ is inscribed in a circle with $m\angle P = 70$, $m\angle Q = 50$, and $m\angle R = 60$. What are the measures of $\widehat{PQ}$, $\widehat{QR}$, and $\widehat{PR}$?

31. **Reasoning** Use the diagram at the right. If you know the values of x and y, how can you find the measure of each numbered angle?

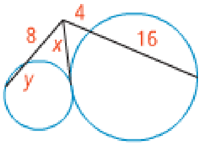


Algebra **Find the values of x and y using the given chord, secant, and tangent lengths. If your answer is not a whole number, round it to the nearest tenth.**

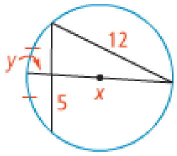
32.



33.



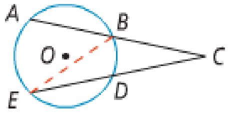
34.



35. Prove Theorem 12-14 as it applies to two secants that intersect outside a circle.
Proof

Given: $\odot O$ with secants $\overline{CA}$ and $\overline{CE}$

Prove: $m\angle ACE = \frac{1}{2}(m\widehat{AE} - m\widehat{BD})$



36. Prove the other two cases of Theorem 12-14. (See Exercise 35.)
Proof

For Exercises 37 and 38, write proofs that use similar triangles.

37. Prove Theorem 12-15, Case II.
Proof

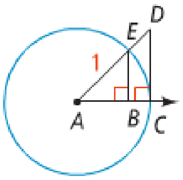
38. Prove Theorem 12-15, Case III.
Proof

39. The diagram at the right shows a *unit circle*, a circle with radius 1.

a. What triangle is similar to $\triangle ABE$?

b. Describe the connection between the ratio for the tangent of $\angle A$ and the segment that is tangent to $\odot A$.

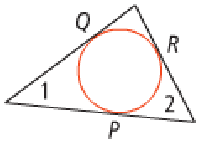
c. The secant ratio is $\frac{\text{hypotenuse}}{\text{length of leg adjacent to an angle}}$. Describe the connection between the ratio for the secant of $\angle A$ and the segment that is the secant in the unit circle.



Challenge **For Exercises 40 and 41, use the diagram at the right. Prove each statement.**

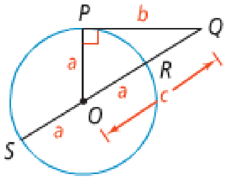
40. $m\angle 1 + m\widehat{PQ} = 180$
Proof

41. $m\angle 1 + m\angle 2 = m\widehat{QR}$
Proof



42. Use the diagram at the right and the theorems of this lesson to prove the
Proof Pythagorean Theorem.

43. If an equilateral triangle is inscribed in a circle, prove that the tangents to the
Proof circle at the vertices form an equilateral triangle.

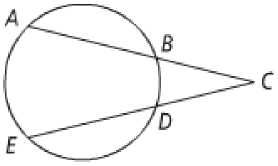


Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

For Exercises 44 and 45, use the diagram at the right.



44. If $BC = 6$, $DC = 5$, and $CE = 12$, find AC.

45. If $m\angle C = 14$ and $m\widehat{AE} = 140$, find $m\widehat{BD}$.

46. The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments of length 6 and 18. What is the length of the altitude to the nearest tenth?

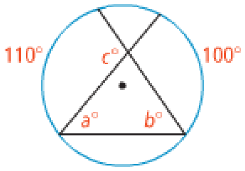
47. A rectangular prism measures 3 cm × 4 cm × 5 cm. The length of the longest side of a similar rectangular prism is 12 cm. What is the volume of the larger prism to the nearest tenth?

Mixed Review

Find the value of each variable.

See Lesson 12-3.

48.



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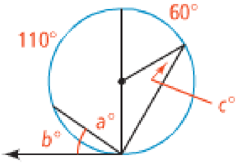
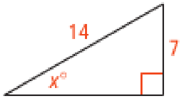


Find the value of *x* to the nearest whole number.

See Lesson 8-3.

50.



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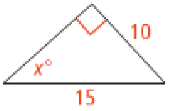

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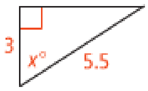
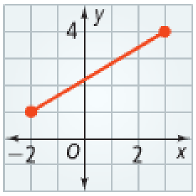


Get Ready! **To prepare for Lesson 12-5, do Exercises 53–55.**

Find the length of each segment to the nearest tenth.

See Lesson 1-7.

53.



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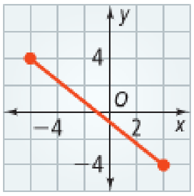

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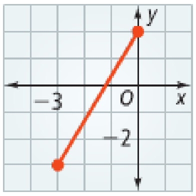
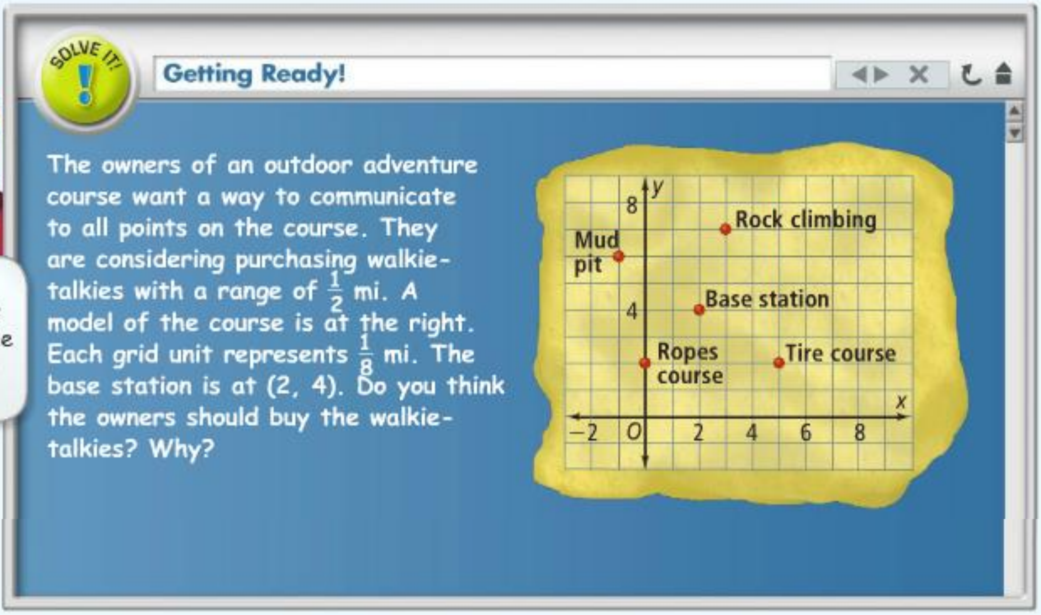


12-5 Circles in the Coordinate Plane

Objectives To write the equation of a circle
To find the center and radius of a circle

I'd like to know that help is on its way if I fall on the course's climbing wall!



Getting Ready!

The owners of an outdoor adventure course want a way to communicate to all points on the course. They are considering purchasing walkie-talkies with a range of $\frac{1}{2}$ mi. A model of the course is at the right. Each grid unit represents $\frac{1}{8}$ mi. The base station is at (2, 4). Do you think the owners should buy the walkie-talkies? Why?

Dynamic Activity
Circles in the Coordinate Plane

Lesson Vocabulary
- standard form of an equation of a circle

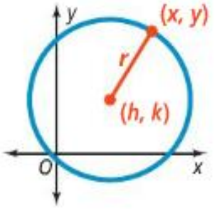
In the Solve It, all of the obstacles lie within or on a circle with the base station as the center. The information from the diagram is enough to write an equation for the circle.

Essential Understanding The information in the equation of a circle allows you to graph the circle. Also, you can write the equation of a circle if you know its center and radius.

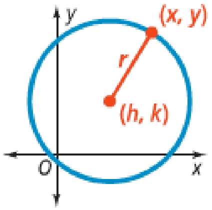
take note

Theorem 12-16 Equation of a Circle

An equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.



Here's Why It Works You can use the Distance Formula to find an equation of a circle with center (h, k) and radius r, which proves Theorem 12-16. Let (x, y) be any point on the circle. Then the radius r is the distance from (h, k) to (x, y).



$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$r = \sqrt{(x - h)^2 + (y - k)^2}$ Substitute (x, y) for (x_2, y_2) and (h, k) for (x_1, y_1).

$r^2 = (x - h)^2 + (y - k)^2$ Square both sides.

The equation $(x - h)^2 + (y - k)^2 = r^2$ is the **standard form of an equation of a circle.** You may also call it the *standard equation* of a circle.

Problem 1 Writing the Equation of a Circle

Plan

What do you need to know to write the equation of a circle? You need to know the values of h, k, and r; h is the x-coordinate of the center, k is the y-coordinate of the center, and r is the radius.

What is the standard equation of the circle with center $(5, -2)$ and radius 7?

$(x - h)^2 + (y - k)^2 = r^2$ Use the standard form of an equation of a circle.

$(x - 5)^2 + [y - (-2)]^2 = 7^2$ Substitute $(5, -2)$ for (h, k) and 7 for r.

$(x - 5)^2 + (y + 2)^2 = 49$ Simplify.

Got It? **1.** What is the standard equation of each circle?

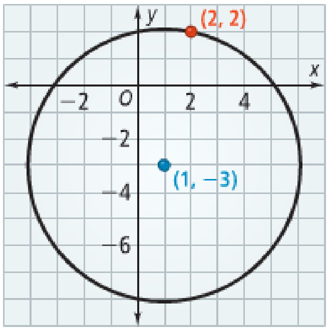
a. center $(3, 5)$; radius 6 **b.** center $(-2, -1)$; radius $\sqrt{2}$

Problem 2 Using the Center and a Point on a Circle

Think

How is this problem different from Problem 1? In this problem, you don't know r. So the first step is to find r.

What is the standard equation of the circle with center $(1, -3)$ that passes through the point $(2, 2)$?



Step 1 Use the Distance Formula to find the radius.

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Use the Distance Formula.

$= \sqrt{(1 - 2)^2 + (-3 - 2)^2}$ Substitute $(1, -3)$ for (x_2, y_2) and $(2, 2)$ for (x_1, y_1).

$= \sqrt{(-1)^2 + (-5)^2}$ Simplify.

$= \sqrt{26}$

Step 2 Use the radius and the center to write an equation.

$(x - h)^2 + (y - k)^2 = r^2$ Use the standard form of an equation of a circle.

$(x - 1)^2 + [y - (-3)]^2 = (\sqrt{26})^2$ Substitute $(1, -3)$ for (h, k) and $\sqrt{26}$ for r.

$(x - 1)^2 + (y + 3)^2 = 26$ Simplify.

Got It? **2.** What is the standard equation of the circle with center $(4, 3)$ that passes through the point $(-1, 1)$?

If you know the standard equation of a circle, you can describe the circle by naming its center and radius. Then you can use this information to graph the circle.

Problem 3 Graphing a Circle Given Its Equation

Communications When you make a call on a cell phone, a tower receives and transmits the call. A way to monitor the range of a cell tower system is to use equations of circles. Suppose the equation $(x - 7)^2 + (y + 2)^2 = 64$ represents the position and the transmission range of a cell tower. What is the graph that shows the position and range of the tower?

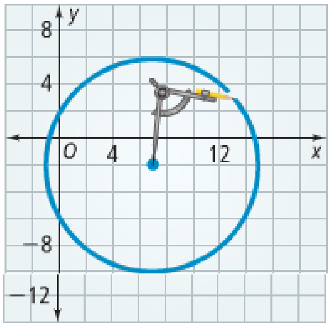
Know	Need	Plan
The equation representing the cell tower's position and range	To draw a graph	Determine the values of (h, k) and r in the equation. Then draw a graph.

$$(x - 7)^2 + (y + 2)^2 = 64$$ Use the standard equation of a circle.

$$(x - \underset{h}{\underset{\uparrow}{7}})^2 + [y - (\underset{k}{\underbrace{-2}})] = \underset{r}{\underset{\uparrow}{8}}{}^2$$ Rewrite to find h, k, and r.

The center is $(7, -2)$ and the radius is 8.

To graph the circle, place the compass point at the center $(7, -2)$ and draw a circle with radius 8.



Got It? 3. a. In Problem 3, what does the center of the circle represent? What does the radius represent?

b. What is the center and radius of the circle with equation $(x - 2)^2 + (y - 3)^2 = 100$? Graph the circle.

Lesson Check

Do you know HOW?

What is the standard equation of each circle?

1. center $(0, 0)$; $r = 4$

2. center $(1, -1)$; $r = \sqrt{5}$

What is the center and radius of each circle?

3. $(x - 8)^2 + y^2 = 9$

4. $(x + 2)^2 + (y - 4)^2 = 7$

Do you UNDERSTAND?

5. What is the least amount of information that you need to graph a circle? To write the equation of a circle?

6. Suppose you know the center of a circle and a point on the circle. How do you determine the equation of the circle?

7. Error Analysis A student says that the center of a circle with equation $(x - 2)^2 + (y + 3)^2 = 16$ is $(-2, 3)$. What is the student's error?

Practice and Problem-Solving Exercises

A Practice

Write the standard equation of each circle. **See Problem 1.**

8. center $(2, -8)$; $r = 9$

9. center $(0, 3)$; $r = 7$

10. center $(0.2, 1.1)$; $r = 0.4$

11. center $(5, -1)$; $r = 12$

12. center $(-6, 3)$; $r = 8$

13. center $(-9, -4)$; $r = \sqrt{5}$

14. center $(0, 0)$; $r = 4$

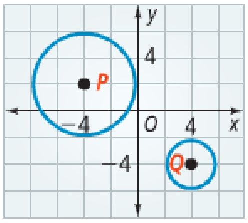
15. center $(-4, 0)$; $r = 3$

16. center $(-1, -1)$; $r = 1$

Write a standard equation for each circle in the diagram at the right. **See Problem 2.**

17. $\odot P$

18. $\odot Q$



Write the standard equation of the circle with the given center that passes through the given point.

19. center $(-2, 6)$; point $(-2, 10)$

20. center $(1, 2)$; point $(0, 6)$

21. center $(7, -2)$; point $(1, -6)$

22. center $(-10, -5)$; point $(-5, 5)$

23. center $(6, 5)$; point $(0, 0)$

24. center $(-1, -4)$; point $(-4, 0)$

Find the center and radius of each circle. Then graph the circle. **See Problem 3.**

25. $(x + 7)^2 + (y - 5)^2 = 16$

26. $(x - 3)^2 + (y + 8)^2 = 100$

27. $(x + 4)^2 + (y - 1)^2 = 25$

28. $x^2 + y^2 = 36$

Public Safety **Each equation models the position and range of a tornado alert siren. Describe the position and range of each.**

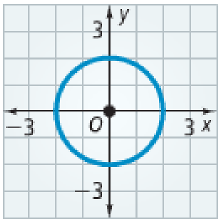
29. $(x - 5)^2 + (y - 7)^2 = 81$

30. $(x + 4)^2 + (y - 9)^2 = 144$

B Apply

Write the standard equation of each circle.

31.



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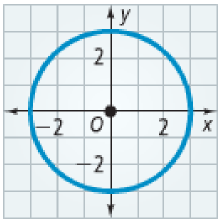

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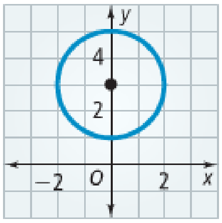

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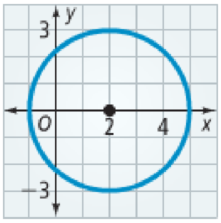

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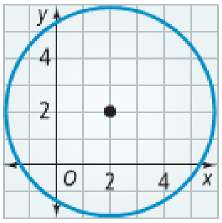

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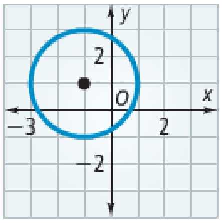


Write an equation of a circle with diameter $\overline{AB}$.

37. $A(0, 0), B(8, 6)$

38. $A(3, 0), B(7, 6)$

39. $A(1, 1), B(5, 5)$

40. **Reasoning** Describe the graph of $x^2 + y^2 = r^2$ when $r = 0$.

Determine whether each equation is the equation of a circle. Justify your answer.

41. $(x - 1)^2 + (y + 2)^2 = 9$

42. $x + y = 9$

43. $x + (y - 3)^2 = 9$

44. **Think About a Plan** Find the circumference and area of the circle whose equation is $(x - 9)^2 + (y - 3)^2 = 64$. Leave your answers in terms of π.

- What essential information do you need?
- What formulas will you use?

45. Write an equation of a circle with area 36π and center $(4, 7)$.

46. What are the x- and y-intercepts of the line tangent to the circle $(x - 2)^2 + (y - 2)^2 = 5^2$ at the point $(5, 6)$?

47. For $(x - h)^2 + (y - k)^2 = r^2$, show that $y = \sqrt{r^2 - (x - h)^2} + k$ or $y = -\sqrt{r^2 - (x - h)^2} + k$.

Sketch the graphs of each equation. Find all points of intersection of each pair of graphs.

48. $x^2 + y^2 = 13$
$y = -x + 5$

49. $x^2 + y^2 = 17$
$y = -\frac{1}{4}x$

50. $x^2 + y^2 = 8$
$y = 2$

51. $x^2 + y^2 = 20$
$y = -\frac{1}{2}x + 5$

52. $(x + 1)^2 + (y - 1)^2 = 18$
$y = x + 8$

53. $(x - 2)^2 + (y - 2)^2 = 10$
$y = -\frac{1}{3}x + 6$

Graphing Calculator **Use a graphing calculator to convince yourself that the given line is not tangent to the circle $x^2 + y^2 = 25$. Explain what you did.**

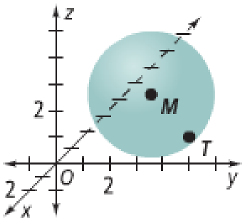
54. $y = -5x + 26$

55. $3x + 5y = 29$

56. **Writing** Why it is not possible to conclude that a line and a circle are tangent by viewing their graphs?

Challenge

57. **Geometry in 3 Dimensions** The equation of a sphere is similar to the equation of a circle. The equation of a sphere with center (h, j, k) and radius r is $(x - h)^2 + (y - j)^2 + (z - k)^2 = r^2$. $M(-1, 3, 2)$ is the center of a sphere passing through $T(0, 5, 1)$. What is the radius of the sphere? What is the equation of the sphere?



58. The concentric circles $(x - 3)^2 + (y - 5)^2 = 64$ and $(x - 3)^2 + (y - 5)^2 = 25$ form a ring. The lines $y = \frac{2}{3}x + 3$ and $y = 5$ intersect the ring, making four sections. Find the area of each section. Round your answers to the nearest tenth of a square unit.

59. **Nautical Distance** A close estimate of the radius of Earth's equator is 3960 mi.

a. Write the equation of the equator with the center of Earth as the origin.

b. Find the length of a 1° arc on the equator to the nearest tenth of a mile.

c. **History** Columbus planned his trip to the East by going west. He thought each 1° arc was 45 mi long. He estimated that the trip would take 21 days. Use your answer to part (b) to find a better estimate.

Standardized Test Prep

SAT/ACT

60. What is an equation of a circle with radius 16 and center $(2, -5)$?

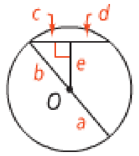
Ⓐ $(x - 2)^2 + (y + 5)^2 = 16$

Ⓑ $(x + 2)^2 + (y - 5)^2 = 4$

Ⓒ $(x + 2)^2 + (y - 5)^2 = 256$

Ⓓ $(x - 2)^2 + (y + 5)^2 = 256$

61. What can you NOT conclude from the diagram at the right?



Ⓕ $c = d$

Ⓖ $c^2 + e^2 = b^2$

Ⓗ $a = b$

Ⓘ $e = d$

Short Response

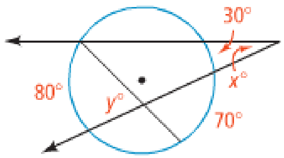
62. Are the following statements equivalent?

- In a circle, if two central angles are congruent, then they have congruent arcs.
- In a circle, if two arcs are congruent, then they have congruent central angles.

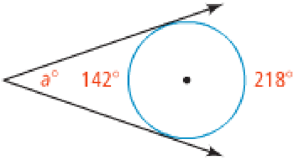
Mixed Review

Find the value of each variable. See Lesson 12-4.

63.



64.



For the given vectors $\vec{a}$ and $\vec{c}$, write the sum $\vec{a} + \vec{c}$ as an ordered pair. See Lesson 8-5.

65. $\vec{a} = \langle -2, 5\rangle$ and $\vec{c} = \langle 8, 7\rangle$

66. $\vec{a} = \langle -3, -4\rangle$ and $\vec{c} = \langle -2, 6\rangle$

67. $\vec{a} = \langle 3, 1\rangle$ and $\vec{c} = \langle 1, 3\rangle$

68. $\vec{a} = \langle 9, -6\rangle$ and $\vec{c} = \langle 2, -1\rangle$

Get Ready! **To prepare for Lesson 12-6, do Exercises 69–71.** See Lessons 1-2 and 1-5.

Sketch each of the following.

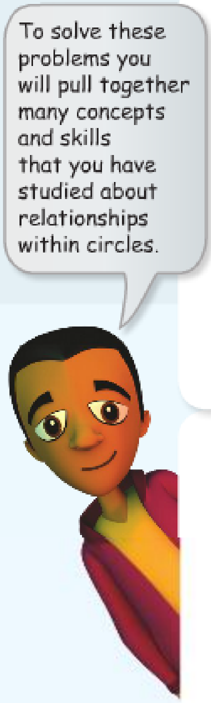
69. the perpendicular bisector of $\overline{BC}$

70. line k parallel to line m and perpendicular to line w, all in plane N

71. $\angle EFG$ bisected by $\overrightarrow{FH}$

12

Pull It All Together

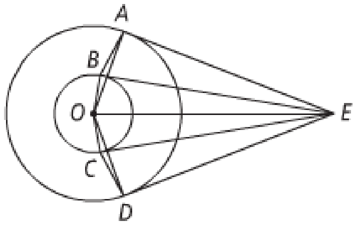

To solve these problems you will pull together many concepts and skills that you have studied about relationships within circles.

BIG idea Reasoning and Proof

You can use triangle congruence theorems to prove relationships among tangents and secants.

Task 1

Four tangents are drawn from *E* to two concentric circles. *A, B, C,* and *D* are the points of tangency. Name as many pairs of congruent triangles as possible and tell how you can show each pair is congruent.

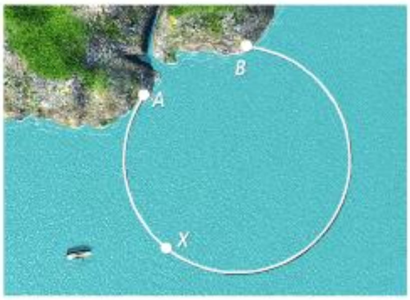


BIG idea Measurement

You can use facts about arcs and angle measures to solve real-world problems.

Task 2

The rocks near the shore between two lighthouses at points *A* and *B* make the waters unsafe. The measure of $\widehat{AXB}$ is 300. Waters inside this arc are unsafe. Suppose you are a navigator on a ship at sea. How can you use the lighthouses to keep the ship in safe waters?



BIG idea Coordinate Geometry

You can use coordinates of the center of a circle and its radius to write an equation for a circle.

Task 3

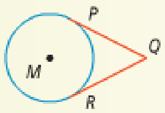
A gardener wants the three rosebushes in her garden to be watered by a rotating water sprinkler. The gardener draws a diagram of the garden using a grid in which each unit represents 1 ft. The rosebushes are at (1, 3), (5, 11), and (11, 4). She wants to position the sprinkler at a point equidistant from each rosebush. Where should the gardener place the sprinkler? What equation describes the boundary of the circular region that the sprinkler will cover?

12 Chapter Review

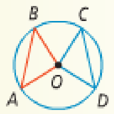
Connecting BIG ideas and Answering the Essential Questions

1 Reasoning and Proof
The measure of an arc equals the measure of its central angle. You can use this angle and arc to prove relationships of other angles and arcs.

Tangents, Chords, and Arcs (Lessons 12-1, 12-2)



$\overline{PQ}$ and $\overline{RQ}$ are tangents to $\odot M$. So $\overline{PQ} \cong \overline{RQ}$.

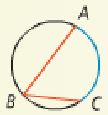


In a circle or congruent circles, if one is true, all are true:
$\angle AOB \cong \angle COD$,
$\overset{\frown}{AB} \cong \overset{\frown}{CD}$, $\overline{AB} \cong \overline{CD}$

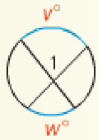
Locus: A Set of Points (Lesson 12-6)
A locus is a set of points, all of which meet a stated condition.

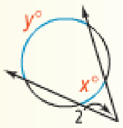
2 Measurement
Segments intersecting circles form angles and intercepted arcs. You can find some missing measures using given information and appropriate formulas.

Angles and Circles (Lessons 12-3, 12-4)



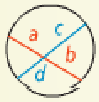
$m\angle B = \frac{1}{2} m\overset{\frown}{AC}$

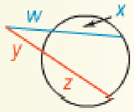


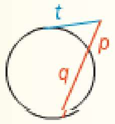


$m\angle 1 = \frac{1}{2}(v + w)$
$m\angle 2 = \frac{1}{2}(y - x)$

Segment Lengths (Lesson 12-4)



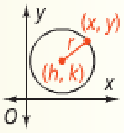




$a \cdot b = c \cdot d$ $(w + x)w = (y + z)y$
$(p + q)p = t^2$

3 Coordinate Geometry
You can use the center and the radius to write an equation of a circle.

Circles in the Coordinate Plane (Lesson 12-5)
$(x - h)^2 + (y - k)^2 = r^2$



Chapter Vocabulary

- chord (p. 771)
- inscribed angle (p. 780)
- intercepted arc (p. 780)
- locus (p. 804)
- point of tangency (p. 762)
- secant (p. 791)
- standard form of an equation of a circle (p. 799)
- tangent to a circle (p. 762)

Use the figure to choose the correct term to complete each sentence.

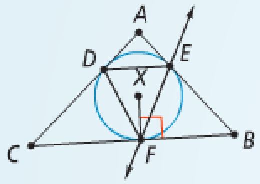
1. $\overleftrightarrow{EF}$ is (*a secant of, tangent to*) $\odot X$.

2. $\overline{DF}$ is a (*chord, locus*) of $\odot X$.

3. $\triangle ABC$ is made of (*chords in, tangents to*) $\odot X$.

4. $\angle DEF$ is an (*intercepted arc, inscribed angle*) of $\odot X$.

5. The set of all points equidistant from the endpoints of $\overline{CB}$ is a (*locus, tangent*).



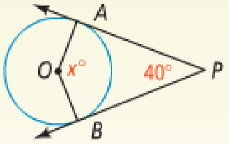
12-1 Tangent Lines

Quick Review

A **tangent** to a circle is a line that intersects the circle at exactly one point. The radius to that point is perpendicular to the tangent. From any point outside a circle, you can draw two segments tangent to a circle. Those segments are congruent.

Example

$\overrightarrow{PA}$ and $\overrightarrow{PB}$ are tangents. Find x.

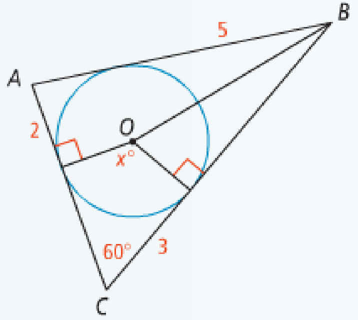


The radii are perpendicular to the tangents. Add the angle measures of the quadrilateral:

$$x + 90 + 90 + 40 = 360$$
$$x + 220 = 360$$
$$x = 140$$

Exercises

Use $\odot O$ for Exercises 6–8.



6. What is the perimeter of $\triangle ABC$?

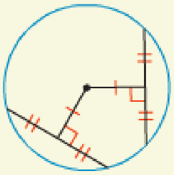
7. $OB = \sqrt{28}$. What is the radius?

8. What is the value of x?

12-2 Chords and Arcs

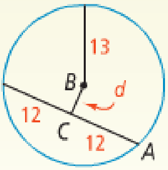
Quick Review

A **chord** is a segment whose endpoints are on a circle. Congruent chords are equidistant from the center. A diameter that bisects a chord that is not a diameter is perpendicular to the chord. The perpendicular bisector of a chord contains the center of the circle.



Example

What is the value of d?

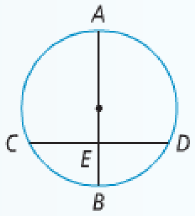


Since the chord is bisected, $m\angle ACB = 90$. The radius is 13 units. So an auxiliary segment from A to B is 13 units. Use the Pythagorean Theorem.

$$d^2 + 12^2 = 13^2$$
$$d^2 = 25$$
$$d = 5$$

Exercises

Use the figure at the right for Exercises 9–11.



9. If $\overline{AB}$ is a diameter and $CE = ED$, then $m\angle AEC =$ _?_.

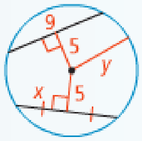
10. If $\overline{AB}$ is a diameter and is at right angles to $\overline{CD}$, what is the ratio of CD to DE?

11. If $CE = \frac{1}{2}CD$ and $m\angle DEB = 90$, what is true of $\overline{AB}$?

Use the circle below for Exercises 12 and 13.

12. What is the value of x?

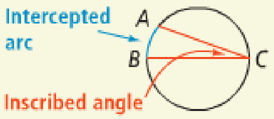
13. What is the value of y?



12-3 Inscribed Angles

Quick Review

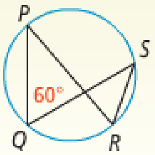
An **inscribed angle** has its vertex on a circle and its sides are chords. An **intercepted arc** has its endpoints on the sides of an inscribed angle, and its other points in the interior of the angle. The measure of an inscribed angle is half the measure of its intercepted arc.



Example

What is $m\widehat{PS}$? What is $m\angle R$?

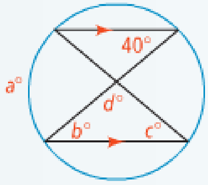
The $m\angle Q = 60$ is half of $m\widehat{PS}$, so $m\widehat{PS} = 120$. $\angle R$ intercepts the same arc as $\angle Q$, so $m\angle R = 60$.



Exercises

Find the value of each variable. Line ℓ is a tangent.

14.



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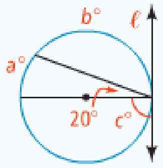

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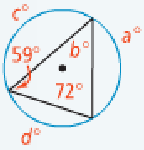

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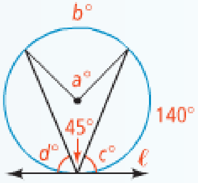
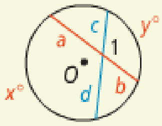


12-4 Angle Measures and Segment Lengths

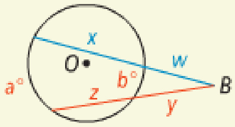
Quick Review

A **secant** is a line that intersects a circle at two points. The following relationships are true:



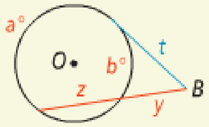
$a \cdot b = c \cdot d$

$m\angle 1 = \frac{1}{2}(x + y)$



$(w + x)w = (y + z)y$

$m\angle B = \frac{1}{2}(a - b)$



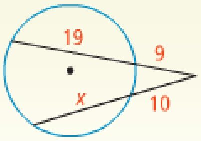
$(y + z)y = t^2$

$m\angle B = \frac{1}{2}(a - b)$

Example

What is the value of x?

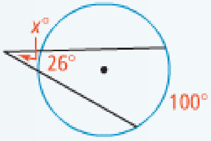
$$(x + 10)10 = (19 + 9)9$$
$$10x + 100 = 252$$
$$x = 15.2$$



Exercises

Find the value of each variable.

18.



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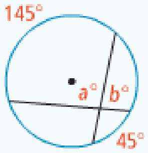

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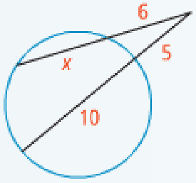

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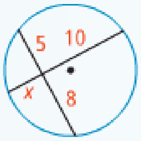
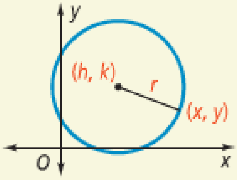


12-5 Circles in the Coordinate Plane

Quick Review

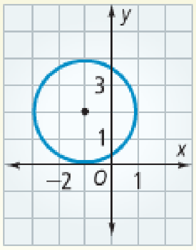
The **standard form of an equation of a circle** with center (h, k) and radius r is

$(x - h)^2 + (y - k)^2 = r^2$.



Example

Write the standard equation of the circle shown.



The center is $(-1, 2)$. The radius is 2.

The equation of the circle is

$(x - (-1))^2 + (y - 2)^2 = 2^2$

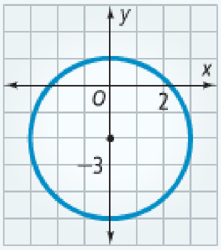
or

$(x + 1)^2 + (y - 2)^2 = 4$.

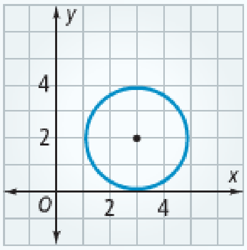
Exercises

Write the standard equation of each circle below.

22.



23.



24. What is the standard equation of the circle with radius 5 and center $(-3, -4)$?

25. What is the standard equation of the circle with center $(1, 4)$ that passes through $(-2, 4)$?

26. What are the center and radius of the circle with equation $(x - 7)^2 + (y + 5)^2 = 36$?

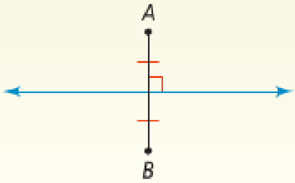
12-6 Locus: A Set of Points

Quick Review

A **locus** is a set of points that satisfies a stated condition.

Example

Sketch and describe the locus of points in a plane equidistant from points A and B.



The locus is the perpendicular bisector of $\overline{AB}$.

Exercises

Describe each locus of points.

27. The set of all points in a plane that are in the interior of an angle and equidistant from the sides of the angle.

28. The set of all points in a plane that are 5 cm from a circle with radius 2 cm.

29. The set of all points in a plane at a distance 8 in. from a given line.

30. The set of all points in space that are a distance 6 in. from $\overline{AB}$.

12 Chapter Test

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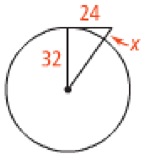
Do you know HOW?

Algebra **For Exercises 1–8, lines that appear tangent are tangent. Find the value of each variable. Round decimals to the nearest tenth.**

1.



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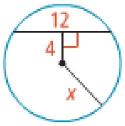

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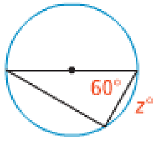

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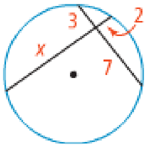

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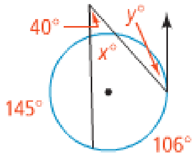

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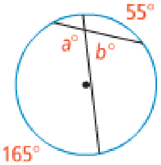

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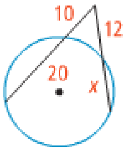

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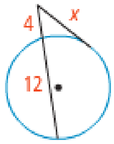
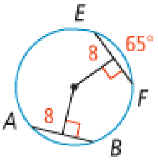


Find $m\widehat{AB}$.

9.



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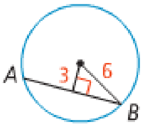


11. Graph $(x + 3)^2 + (y - 2)^2 = 9$. Then label the center and radius.

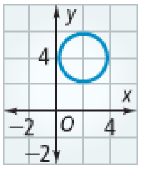
12. Write an equation of the circle with center (3, 0) that passes through point (−2, −4).

13. What is the graph of $x^2 + y^2 = 0$?

14. Write an equation for the locus of points in the coordinate plane that are 4 units from (−5, 2).

Write the standard equation of each circle.

15.



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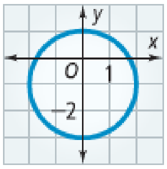


Sketch each locus on a coordinate plane.

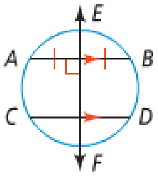
17. the set of all points 3 units from the line $y = -2$

18. the set of all points equidistant from the axes

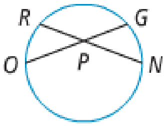
Do you UNDERSTAND?

19. **Writing** What is special about a rhombus inscribed in a circle? Justify your answer.

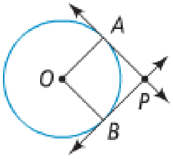
20. **Reasoning** $\overleftrightarrow{EF}$ is the perpendicular bisector of chord $\overline{AB}$, and $\overline{CD} \parallel \overline{AB}$. Show that $\overleftrightarrow{EF}$ is the perpendicular bisector of chord $\overline{CD}$.



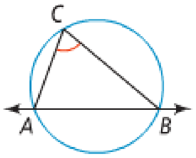
21. **Error Analysis** A student says that $\angle RPO \cong \angle NPG$ in this circle, since they are vertical angles, and thus $\widehat{RO} \cong \widehat{NG}$. Why is this incorrect?



22. **Reasoning** $\overleftrightarrow{PA}$ and $\overleftrightarrow{PB}$ are tangent to $\odot O$. PA is equal to the radius of the circle. What kind of quadrilateral is $PAOB$? Explain.



23. **Reasoning** A secant line passes through a circle at points A and B. Point C is also on the circle. Describe the locus of points P that satisfy these conditions: P and C are on the same side of the secant, and $m\angle APB = m\angle ACB$.



End-of-Course Assessment

Multiple Choice

Read each question. Then write the letter of the correct answer on your paper.

1. What is the standard equation of a circle with center (−2, 0) and radius 4?

Ⓐ $(x - 2)^2 + y^2 = 4$ Ⓒ $x^2 + (y - 2)^2 = 2^2$

Ⓑ $(x + 2)^2 + y^2 = 16$ Ⓓ $x^2 + (y + 2)^2 = 16$

2. Which of the following could serve as a counterexample to the assertion below?

If each pair of opposite sides of a quadrilateral are equal in length and parallel, then the quadrilateral is a square.

Ⓕ kite   Ⓗ square

Ⓖ rhombus   Ⓘ trapezoid

3. $\overleftrightarrow{AB}$ is tangent to $\odot C$ at point B. Which of the following can you NOT conclude is true?

Ⓐ $m\angle CAB < m\angle ACB$

Ⓑ $AB^2 + BC^2 = AC^2$

Ⓒ $\angle CAB$ and $\angle ACB$ are complements.

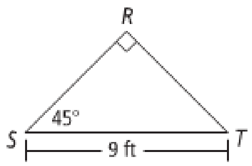
Ⓓ $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

4. The lengths of two sides of a triangle are 4 cm and 8 cm. Which distance could be the length of the third side?

Ⓕ 4 cm   Ⓗ 12 cm

Ⓖ 8 cm   Ⓘ 16 cm

5. What is the area of $\triangle RST$ in square feet?

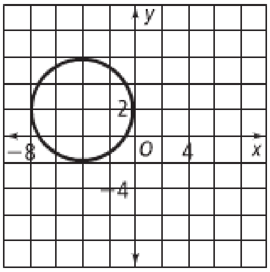


Ⓐ $9\sqrt{2}$ Ⓒ 40.5

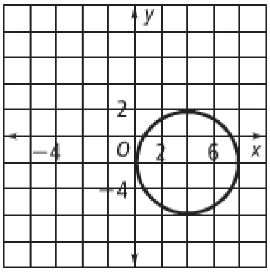
Ⓑ 20.25   Ⓓ $81\sqrt{2}$

6. Which graph represents the equation $(x - 4)^2 + (y + 2)^2 = 16$?

Ⓕ



Ⓖ



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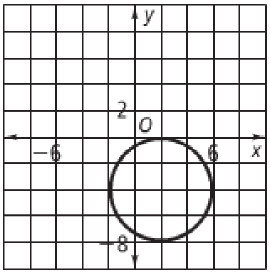

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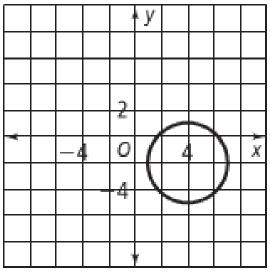
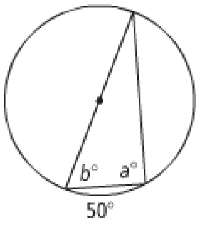


7. What are the values of a and b in the figure below?



Ⓐ $a = 90$ and $b = 130$

Ⓑ $a = 50$ and $b = 130$

Ⓒ $a = 90$ and $b = 65$

Ⓓ $a = 65$ and $b = 65$

8. A rancher wants to enclose a pen for new calves using 60 ft of fencing. Which shape uses all of the fencing and encloses the greatest area?

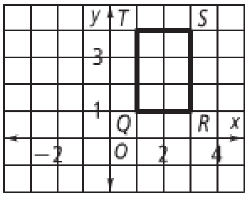
Ⓕ a rectangle that is 12 ft by 18 feet

Ⓖ a square 15 ft on each side

Ⓗ a rectangle that is 20 ft by 30 feet

Ⓘ a circle with an 8-ft radius

9. Figure $QRST$ is shown in the coordinate plane. Which transformation creates an image with a vertex at the point $(-2, 1)$?



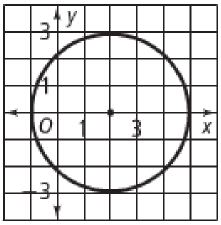
Ⓐ Rotate figure $QRST$ by 90° around R.

Ⓑ Reflect figure $QRST$ across the line $y = 1$.

Ⓒ Reflect figure $QRST$ across the line $x = 1$.

Ⓓ Rotate figure $QRST$ by 90° around Q.

10. What is the standard equation of the circle?



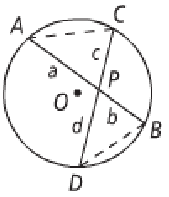
Ⓕ $x^2 + (y - 2)^2 = 9$

Ⓖ $(x + 2)^2 + y^2 = 3$

Ⓗ $(x - 2)^2 + y^2 = 9$

Ⓘ $(x - 2)^2 + y^2 = 3$

11. Amy is trying to prove that $a \cdot b = c \cdot d$ using $\odot O$ below by first proving that $\triangle APC \sim \triangle DPB$. Which similarity theorem or postulate can Amy use?



Ⓐ Side-Side-Side Similarity Theorem

Ⓑ Side-Angle-Side Similarity Theorem

Ⓒ Angle-Angle Similarity Postulate

Ⓓ None of these

12. All four angles of a quadrilateral have the same measure. Which statement is true?

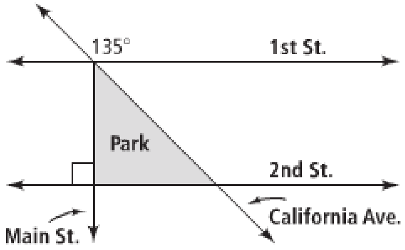
Ⓕ All four sides of the quadrilateral must have the same length.

Ⓖ All four angles of the quadrilateral are acute.

Ⓗ Opposite sides of the quadrilateral are parallel.

Ⓘ The quadrilateral must be a square.

13. A triangular park is bordered by three streets, as shown in the map below.



If 1st Street and 2nd Street are parallel, what are the measures of the three angles of the park?

Ⓐ 90, 45, 45

Ⓑ 90, [illegible], 55

Ⓒ 90, 25, 65

Ⓓ 135, 25, 10

14. What are the coordinates of the midpoint of $\overline{QS}$ with endpoints $Q(-2, -5)$and $S(3, -8)$?

Ⓕ $(-2.5, 1.5)$

Ⓖ $(0.5, -6.5)$

Ⓗ $(-2.5, 6.5)$

Ⓘ $(0.5, 1.5)$